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VIBRATION ANALYSIS OF PIPING SYSTEMS

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VIBRATION ANALYSIS OF PIPING SYSTEMS

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George E. Fink

VIBRATION ANALYSIS OF PIPING SYSTEMS

by

George E. Fink
Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1964

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This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

from the

United States Postgraduate School

ABSTRACT

A digital computer program is presented capable of determining the in- and/or out-of-plane vibration frequencies of a single plane piping system. The analysis is made utilizing the method of transfer matrices, a discussion of which is included. The program accepts branched systems with or without intermediate supports. The effects of both shear deflection and rotational inertia are considered although either or both may be neglected at the user's discretion. Distributed mass is generally treated as such, although provision is made for alternate treatment by lumping mass.

Several typical piping systems are analyzed using these various program options, and the results obtained are compared.

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CHAPTER I

INTRODUCTION

1.1 General Remarks

The dynamic behavior of piping systems, particularly their response to periodic and/or transient excitation, is a subject of considerable interest and growing pertinence in the design of such systems. The reasons for this are readily discernable when the unprecedented reliability demanded of nuclear power plant piping, or the consequence of a piping system failure in an orbiting space vehicle, are considered. An increasing number of piping systems are being designed, of which, like those mentioned above, a long service life is demanded, and to which access is limited or completely denied once the system is placed in service. Clearly, no factor which might contribute to the failure of such a system can be ignored. Fatigue brought about by periodic excitation at or near a system resonance is one such factor. Catastrophic failure caused by a periodic disturbance of short duration and high amplitude, such as an earthquake, is another.

With regard to the latter case, it has come to the writer's attention since beginning this work, that Japanese engineers are giving considerable and close

attention to this particular problem. Their first step in approaching it is to determine the natural frequencies of the system of interest.

Fortunately, or perhaps fostered by necessity, development of the "tools" needed to carry out accurate, reliable, and practicable vibration analyses of complex systems, namely analysis by matrix methods with the aid of high speed digital computers, has kept pace with the increasing demand for their employment.

1.2 Scope of Work Presented

A method for determining, analytically, the in- and out-of-plane undamped natural frequencies of planar piping systems has been developed and is presented herein. It employs the method of "Transfer Matrices"[1] to construct an accurate mathematical system model and determine system natural frequencies, and a digital computer to carry out the inherent voluminous numerical computations. VIPIPE, the digital computer program presented in appendices A through C constitutes the embodiment of the method.

---

[1]The theory and method of Transfer Matrices is discussed in sufficient depth for the purposes of this report in chapter 2. A more comprehensive and detailed treatment of the method may be found in ref. 1.

This method is a first but important step toward the ultimate objective of analysis of three dimensional systems of unrestricted topology.

Attention has been confined to the two dimensional or planar case because the theory underlying the treatment of three dimensional systems using Transfer Matrices is not presently available in the literature, and because it is felt that the need for, and utility of, a treatment of the two dimensional case fully warrants the development.

Although in its present state of development, the theory of Transfer Matrices permits the consideration of any planar piping configuration, certain restrictions are imposed on the configurations which may be treated by the method herein presented, by the limitations of the program VIPIPE. These limitations are discussed in detail in section 1.3.

The philosophy of VIPIPE has sufficient generality to make the program readily adaptable to consideration of the three dimensional case as soon as the underlying theory is developed.

1.3 Assumptions and Limitations

The mathematical models used to describe the elastic properties of a system and its components are faithful within the framework of the following considerations:

a. The material of which each component is made is considered to be homogeneous, isotropic, and linearly elastic.

b. The system is conservative (no damping).

c. Distributed mass is treated as such except in curved sections for which a lumped mass model is used. For curved sections, sufficient subsectioning is carried out so as to reduce error to an economically[1] acceptable level.

d. Spring hangers are considered to consist of from one to three mutually perpendicular linear springs, and from one to three torsional springs which act about mutually perpendicular axes. When a spring hanger consists of both linear and torsional springs, the linear springs are normally considered to act along the same mutually perpendicular axes that the torsional spring act about. A more general case may be treated with relative ease and is a subject of discussion in appendix B, section B.96.

Although the theory would permit determining any number of natural frequencies for any planar system having any number of components, branches, and hangers, program VIPIPE, described herein, is limited to the treatment of systems having a maximum of fifty components,

---

[1] Appendix B, section B.93.

twenty branches, and twenty hangers. Further, only single filament branches from a main member may be treated, i.e., all branches must emanate from the same continuous member, and a branch may not emanate from a branch.

1.4 Notation

$[\ ]$	matrix
x,y,z	cartesian right-handed coordinate system
u,v,w	displacements in the x,y,z directions respectively
δ, ψ, ϑ	rotations about the x,y,z axes respectively
d	displacement vector
E	Young's modulus
EJ	bending stiffness
G	shear modulus
GJ_t	torsional stiffness
G	coordinate transformation matrix
I	unit matrix
J_t	polar second moment of area
L	length
M	bending moment
N	normal force
O	null matrix
P	internal force vector
R_1, R_2	square submatrices
S	spring matrix

T	torque
U	transfer matrix
u_{ij}	transfer matrix element
V	shear force
v	unknown vector
Z	state vector
L,R	subscripts, left and right side of components respectively

CHAPTER II

TRANSFER MATRICES

2.1 The State Vector and Transfer Matrix

a. State Vector The state vector at a point i of an elastic system is a column matrix having sufficient elements to completely describe the instantaneous displacement and/or change in orientation of that point from its quiescent position, as well as the force and moment system existing in the member at the same point at the same instant, i.e., those forces and moments which, if a cutting plane were passed through the point at the instant of interest, would have to be applied to each cut face to prevent relative motion between them.

For perfectly general structural systems the notion of a state vector is a rather sophisticated concept. In any properly constituted state vector however the number of elements pertaining to forces and moments is equal to the number pertaining to displacement.

In the case of a planar piping system, the state vector has six elements, three pertaining to displacement, and three pertaining to force. The state vectors for the in- and out-of-plane cases[1] respectively, as used

[1] Henceforth the in-plane case will be referred to as Case 1, the out-of-plane case as Case 2.

in VIPIPE, were chosen to be

$$Z_{ip}= \begin{bmatrix} u \\ v \\ \vartheta \\ M_z \\ V_y \\ N \end{bmatrix} \qquad Z_{oop}= \begin{bmatrix} \delta \\ T \\ -w \\ \psi \\ M_y \\ V_z \end{bmatrix} \qquad (2.1\text{-}1)$$

The elements of these state vectors are the magnitudes of vector quantities of the same name which derive their orientations from those of the x,y,z axes of a local right-handed cartesian coordinate system (fig. 2.1-1), associated with the particular point of interest.

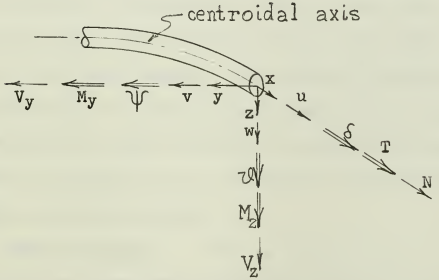

Fig. 2.1-1 Coordinate system and vectorial representation of state vector elements. (Interpret double shaft arrows by the right-hand rule.)

These local coordinate systems are constructed such that the plane of the quiescent system and the xy plane of the coordinate system coincide. The local x axis is coincident with the local tangent to the quiescent system centroidal axis. No reference is made to a common fixed coordinate system. In establishing the coordinate system for the purpose of employing VIPIPE, the longest continuous run of pipe, or the one from which all branches emanate if they are not one and the same, is designated as the main member, one end of which is chosen as the starting point. This end is called the left end and its state vector is symbolized Z_L . The positive x direction is taken as away from the "left" end and toward the "right" end along the centroidal axis of the main member. The positive z direction is perpendicular to the quiescent plane of the system and invariant, and this direction is called downward. The y axis is chosen so as to complete a right-handed cartesian coordinate system.

b. Transfer Matrix Let us consider the relationship between the state vectors at the left and right ends of a massless straight section of pipe. We begin by considering the situation occurring in Case 2 in which both bending and twisting (about the centroidal axis) occur. The relationships between the left and right

end displacements are given by:

$$\delta_R = \delta_L + \frac{TL}{GJ_t} \qquad (2.1\text{-}2)$$

$$-w_R = -w_L + \psi_L L + M_L \frac{L^2}{2EJ} + V_L \frac{L^3}{6EJ} \qquad (2.1\text{-}3)$$

$$\psi_R = \psi_L + M_L \frac{L}{EJ} + V_L \frac{L^2}{2EJ} \qquad (2.1\text{-}4)$$

The force relationships are given by:

$$T_R = T_L \qquad (2.1\text{-}5)$$

$$M_R = M_L + V_L L \qquad (2.1\text{-}6)$$

$$V_R = V_L \qquad (2.1\text{-}7)$$

or, in matrix notation

$$\begin{bmatrix} \delta \\ T \\ -w \\ \psi \\ M_y \\ V_z \end{bmatrix}_R = \begin{bmatrix} 1 & \frac{L}{GJ_t} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & \frac{L^2}{2EJ} & \frac{L^3}{6EJ} \\ 0 & 0 & 0 & 1 & \frac{L}{EJ} & \frac{L^2}{2EJ} \\ 0 & 0 & 0 & 0 & 1 & L \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \delta \\ T \\ -w \\ \psi \\ M_y \\ V_z \end{bmatrix}_L$$

The six-by-six matrix relating the state vectors of the left and right end of the pipe is termed a transfer matrix. It is the transfer matrix of the elastomechical system constituted by the pipe and its boundary conditions. Similar six-by-six transfer matrices may be developed for point masses, curved sections of pipe,

and elbows, etc. Many of these are contained in the transfer matrix catalog included in ref. 1[1] and are used in VIPIPE.

Since the application of interest is to the determination of the natural frequencies of piping systems, it would be instructive at this point to illustrate the origin and nature of frequency dependence in transfer matrices. To that end, let us construct the transfer matrix of a point mass, again considering Case 2. To begin, note that throughout the application considered herein we are concerned with the isochronous vibration of a system at a single frequency ω. The tacit assumption has been made that all quantities varying with time, i.e., all of the elements of the state vectors, do so with the form; constant $\cdot \cos\omega t$, and have exhibited only the constant or amplitude, while suppressing the variable factor.

The transfer matrix of a point mass may be constructed by noting that deflection, twist, slope, and moment are constant and continuous across the zero length section which a point mass constitutes. The vibrating mass does introduce a discontinuity in shear however, which may be determined by considering the free body diagram of

[1] See App. F.

Fig. 2.1-2, and the following relationships;

$$w(t) = w \cdot \cos\omega t$$

$$\frac{d^2 w(t)}{dt^2} = -w\omega^2 \cos\omega t \qquad (2.1\text{-}9)$$

From F=ma:

$$V_{zR}\cos\omega t - V_{zL}\cos\omega t = m\frac{d^2 w(t)}{dt^2} \qquad (2.1\text{-}10)$$

and substituting (2.1-9) into (2.1-10) we have;

$$V_{zR} = V_{zL} + m\omega^2(-w) \qquad (2.1\text{-}11)$$

and are ready to construct the transfer matrix U_P of a point mass. Hence

$$U_P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & m\omega^2 & 0 & 0 & 1 \end{bmatrix} \qquad (2.1\text{-}12)$$

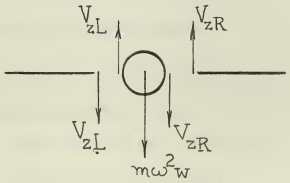

Fig. 2.1-2 Free body diagram of a point mass

Consider now an elastomechanical system, such as that shown in Fig. 2.1-3, made up of many components connected end-to-end, each of which, as should now be apparent, may be characterized by a unique transfer matrix. Subscripting the

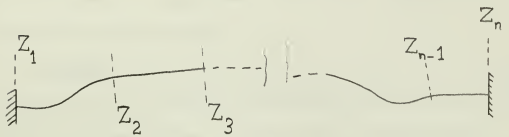

Fig. 2.1-3 An elastomechanical system

transfer matrices to reflect the subscripts assigned to the state vectors which describe the force systems and displacements at the ends of each component, we may write the following matrix equations;

$$Z_2=U_{1,2}\cdot Z_1,\ Z_3=U_{2,3}\cdot Z_2,\cdots,Z_n=U_{n-1,n}\cdot Z_{n-1} \qquad (2.1\text{-}13)$$

Note that successive state vectors may be eliminated, by substituting for each its matrix product equivalent, until finally the matrix relationship

$Z_n=U_{n-1,n}\cdots U_{2,3}\cdot U_{1,2}\cdot Z_1$ remains. The matrix product, $U_{n-1,n}\cdots U_{3,2}\cdot U_{2,1}$, of the component transfer matrices is itself an m-by-m matrix which relates the final to the initial state vector of the system. It is, in fact, the transfer matrix of the system.

2.2 Elastic Intermediate Conditions and Branched Systems

Piping systems frequently include branches, and are supported intermittently along their length by hangers. These branches and hangers usually have a very significant effect on the dynamic behavior of a system. Their effects may be considered within the framework of the transfer matrix method.

a. Hangers It is clear that the effect of a hanger on a system is to introduce discontinuities in the internal forces in the system at the point of attachment.

Consider a spring hanger having the torsional spring constants CTX about the x axis, and CTY about the y axis, and the linear spring constant CLX in the x direction. For a Case 2 situation, the equations relating the state vector elements on either side of the point of attachment are:

$$\delta_R = \delta_L$$

$$T_R = T_L + CTX \cdot \delta$$

$$-w_R = -w_L$$

$$\psi_R = \psi_L$$

$$M_R = M_L + CTY \cdot \psi$$

$$V_R = V_L + CLX \cdot w$$

or in matrix notation;

$$\begin{bmatrix} \delta \\ T \\ -w \\ \psi \\ M_y \\ V_Z \end{bmatrix}_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ CTX & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & CTY & 1 & 0 \\ 0 & 0 & -CLX & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta \\ T \\ -w \\ \psi \\ M_y \\ V_Z \end{bmatrix}_L$$

It should be apparent at this point that a rigid intermediate support, i.e., one having one or more infinitely stiff springs, may not be considered using this method. Rigid supports may be approximated however by using very large values for appropriate spring constants. A value of 100,000 lbs./in. or in.-lb./ rad. as applicable would be an adequate approximation in most instances, and it does not introduce computational difficulties.

b. Branches The joining of a branch to a system has an effect similar to that of a spring hanger on the force system in the main member at the point of joining. In fact, it "looks" exactly like a spring hanger to the main member except, as will shortly be seen, its parameters depend on frequency. Discontinuities are introduced in the forces whereas the main member deflections on each side of the joint are the same, and identical to those of the branch at the same point.

Consider a Case 1 situation and system shown in Fig. 2.2-1.

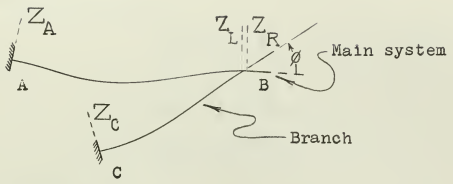

Fig. 2.2-1 Main system with a branch

In the free body diagram, Fig. 2.2-2a, of the joint of the system of Fig. 2.2-1, the force systems acting on the joint are expressed in terms of their own coordinate systems.[1] By a suitable coordinate transformation it is possible to express the force system of the end of the branch in terms of the main

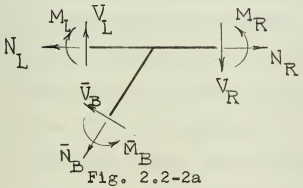

Fig. 2.2-2a

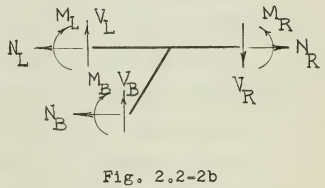

Fig. 2.2-2b

member coordinate system. Such a situation is shown in Fig. 2.2-2b.

---

[1]The symbol $\bar{}$ refers to the fact that the vector elements are expressed in terms of the coordinates of the branch.

Equilibrium considerations and the orthogonality of the force and shear vectors permit us to write the following equations:

$$M_R = M_L + M_B$$

$$V_R = V_L + V_B$$

$$N_R = N_L + N_B$$

or in matrix form

$$\begin{bmatrix} M \\ V \\ N \end{bmatrix}_L + \begin{bmatrix} M \\ V \\ N \end{bmatrix}_B = \begin{bmatrix} M \\ V \\ N \end{bmatrix}_R \qquad (2.2\text{-}0)$$

Bearing in mind the continuity of displacements at a joint, it is clear that the main member state vector Z_R can be obtained by simply adding the force components of the branch state vector Z_B, expressed in the coordinate system of the main member, to the force components of the main member state vector Z_L. What follows is the derivation of a suitable expression for the force components of the branch state vector. The main member state vector Z_L is obtained easily from the state vector Z_A and the transfer matrices of the intervening main member components.

At point C (Fig. 2.2-1) there are three zero and three non zero elements in the state vector Z_C.[1]

[1] See Appendix B, section B.92 for treatment of end conditions which do not meet the requirement for three zero and three non zero elements.

The column matrix of the non zero components is symbolized $\bar{v}$. After relating the branch state vector at B to that at C by the transfer matrix of the branch (obtained in the usual manner from the transfer matrices of its components), and eliminating those columns of the branch transfer matrix which would otherwise be multiplied by the zeros of Z_C, we obtain the matrix equation

$$\begin{bmatrix} \bar{d}_B \\ \hline \bar{P}_B \end{bmatrix} = \begin{bmatrix} R \\ 1 \\ \hline R_2 \end{bmatrix} \circ \bar{v} \qquad (2.2\text{-}1)$$

wherein the state vector $\bar{Z}_B$ has been reordered and partitioned such that $\bar{d}_B$ denotes a sub vector containing the elements of $\bar{Z}_B$ pertaining to deflection, and $\bar{P}_B$ a sub vector containing those pertaining to force. Expansion of equation 2.2-1 gives

$$\bar{d}_B = R_1 \circ \bar{v} \qquad (2.2\text{-}2)$$

and

$$\bar{P}_B = R_2 \circ \bar{v} \qquad (2.2\text{-}3)$$

By substituting $R_1^{-1} \circ \bar{d}_B = \bar{v}$ in equation 2.2-3 we obtain

$$\bar{P}_B = R_2 \circ R_1^{-1} \circ \bar{d}_B \qquad (2.2\text{-}4)$$

relating the displacements at B to the forces in the branch at B. The matrix product $R_2 \circ R_1^{-1}$ is called a spring matrix because of the function that it serves.

Although any consistant system will do, it is convenient here to establish a particular method for measuring the angle at which a branch joins the main member. The one defined is consistant with that used in VIPIPE. Accordingly, the angle at which a branch joins the main member is measured counter-clockwise from the x axis of the main member as it is oriented immediately before the intersection, to the x axis of the branch as it is oriented at the point of intersection. The angle $\emptyset$ in Fig. 2.2-1 is drawn in conformance with this definition.

From the matrix relationships

$$\bar{d}_B = G_1 \circ d_B$$

and

$$P_B = G_2 \circ \bar{P}_B$$

where for a planar piping system

$$G_1 = \begin{bmatrix} \cos\emptyset & -\sin\emptyset & 0 \\ \sin\emptyset & \cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad G_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\emptyset & -\sin\emptyset \\ 0 & \sin\emptyset & \cos\emptyset \end{bmatrix}$$

the spring matrix (symbolized S)

$$S = G_2 \circ R_2 \circ R_1^{-1} \circ G_1$$

is obtained which relates the force and displacement elements of the branch transfer matrix Z_B (expressed in terms of the coordinates of the main member). Thus

$$P_B = S \circ d_B$$

It is now a simple matter to construct the transfer matrix relating the main member state vectors on either side of the joint. Recall from equation 2.2-0 that

$$P_R = P_L + P_B = P_L + S \cdot d_B$$

since

$$d_R = d_L = d_B ,$$

$$Z_R = \left[\begin{array}{c|c} I & 0 \\ \hline S & I \end{array}\right] \cdot \left[\begin{array}{c} d \\ \hline P \end{array}\right]_L \quad \text{or} \quad Z_R = U_P \cdot Z_L$$

where we use the symbol U_p to denote the point matrix which has just been derived.

Thus a branch is treated as a point transfer matrix in constructing the transfer matrix of the system.

2.3 Mode Frequencies and the Frequency Determinant

A principal application of the method of transfer matrices, and the one with which we are concerned, is to the vibration analysis of elastomechanical systems for the purpose of obtaining their natural frequencies.

When transfer matrices are constructed for systems having mass, certain of their elements are frequency dependent (recall the transfer matrix of a point mass derived earlier). Proper application of the system boundary conditions to these transfer matrices yields a matrix of order $n/2$ (from a transfer matrix of order n) which can be used to obtain the natural frequencies of a

system. Its determinant is termed the frequency determinant of the system. The concept is best described by an example.

Consider a case 2 situation wherein a system has a system transfer matrix U_S and boundary conditions embodied in its left and right end state vectors Z_L and Z_R. Assume that the left end is fixed and that the right end is free. The matrix equation relating the left and right end state vectors is

$$\begin{bmatrix} \delta \\ 0 \\ -w \\ \psi \\ 0 \\ 0 \end{bmatrix}_R = \begin{bmatrix} u_{1,1} & \cdot & \cdot & \cdot & \cdot & u_{1,6} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ u_{6,1} & \cdot & \cdot & \cdot & \cdot & u_{6,6} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ T \\ 0 \\ 0 \\ M_y \\ V \end{bmatrix}_{Z_L}$$

from which may be constructed the matrix relationship

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_R = \begin{bmatrix} u_{2,2} & u_{2,5} & u_{2,6} \\ u_{5,2} & u_{5,5} & u_{5,6} \\ u_{6,2} & u_{6,5} & u_{6,6} \end{bmatrix} \cdot \begin{bmatrix} T \\ M_y \\ V \end{bmatrix}_{Z_L} \qquad (2.3\text{-}1)$$

Clearly, a non-trivial solution requires that the third order submatrix of equation 2.3-1 must vanish. The determinant of the submatrix is the system frequency determinant mentioned above. The frequencies for which its value is zero are the natural frequencies of the system described by the transfer matrix U_S and the boundary condition state vectors, Z_L and Z_R, from which it was derived.

2.4 The Digital Computer and the Method of Transfer Matrices

Even for a relatively simple system, the elements of the frequency determinant become unwieldly polynomials in radian frequency. Determination of the roots of the equation resulting from an expansion of the frequency determinant is usually quite impractical and recourse must be made to a numerical solution. In a numerical solution the frequency determinant is constructed and evaluated for selected frequencies over a range of interest. Its value is plotted as a function of frequency and the natural frequencies of the system which it represents are the zeros of the plot (Fig. 2.3-1).

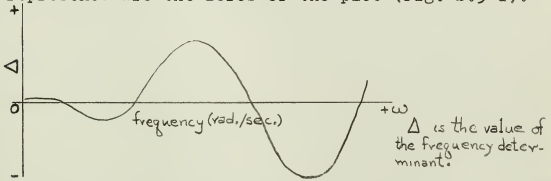

Fig. 2.3-1 Frequency determinant as a function of frequency

Use of a digital computer permits rapid and accurate analysis of complex systems. Program VIPIPE presented in Appendices A through C, for instance, will accept for analysis planar piping systems having up to fifty components including as many as twenty branches and spring hangers. It treats distributed mass as such and considers the effects of shear deflection and rotational inertia. In a typical analysis, the first five

natural frequencies were found for a system having thirteen components, including two spring hangers and two branches, at the cost of five minutes of machine time on a CDC 1604 computer.

It is through the use of high speed digital computers, that the full potential of the method of transfer matrices in this application as well as in others[1] may be realized.

[1]See ref. 1 for a comprehensive treatment of applications of the method of transfer matrices to other problems in elastomechanics.

CHAPTER III

DISCUSSION

3.1 Accuracy of Method

Confidence in the accuracy of the method was established by using it to determine the natural frequencies of several systems for which these frequencies could be obtained by other means, i.e., by means of either a closed form analytical solution or by recourse to the literature [2][1]. The average difference between system natural frequencies obtained using VIPIPE and those obtained by analytical means was found to be less than .0064%, while the average difference between system natural frequencies obtained using VIPIPE and those obtained from the literature was found to be less than .5%.

Details of the systems analyzed and the results obtained may be found in Appendix E, section E.21. The analyzed systems consisted of single spans of straight pipe having various combinations of boundary conditions.

The published papers of E. Voltera [4] give methods for determining the natural frequencies of curved filaments, but the methods are sufficiently involved as to make their application in obtaining some comparison frequencies a major undertaking.

[1]Numbers in brackets pertain to bibliography.

Time precluded assumption of the task. The literature [2,3] does provide means for computing the fundamental frequency of partial rings with fixed ends. The difference between the fundamental frequencies determined using VIPIPE and those computed using the method of the literature, was found to be less than .3% for in-plane, and 2.3% for out-of-plane vibrations.[1] Details of the system analyzed and the results obtained may be found in Appendix E, section E.24.

3.2 Integrity of Solution

Confidence in the integrity of the method was established by conducting "mirror image" analyses of almost every configuration considered. Construction of the transfer matrix of a system is directional in nature, in that an end of the system is chosen as a starting point, and the transfer matrix of the system is developed by premultiplying the incomplete system transfer matrix by the transfer matrix of each system component in the order that each component is encountered in tracing the system from that starting point. It follows therefore that an error in the structure of any of the component transfer matrices, or in the method or order of their combination in constructing the system transfer matrix, would reveal itself in the form

[1] See footnote to section E.24 of Appendix E.

of inconsistent results if the system were analyzed twice, beginning the first time from one end, the second time from the other end. Such a dual analysis is referred to herein as a "mirror image" check.

In general, the disagreement between natural frequencies obtained for a system and for its mirror image was found to be less than .0003%.

An added check on solution integrity was obtained by analyzing a single branch system treating first one member then another as the branch. The average difference here was on the order of .027% for in-plane vibration, and .007% for out-of-plane vibration.

Details of the systems analyzed and of the results obtained may be found in Appendix E, sections E.31 and E.32.

3.3 Computer Imposed Limitations

The number of natural frequencies that may be found for a given system using VIPIPE, is a function of the floating point significant figure capacity of the computer used. The significant figure capacity required to solve for a given number of natural frequencies of any system is a function of the characteristics the system; and for all systems, the significant figure capacity required increases with the number of the mode sought. Thus, for example, for a given system, the number of significant figures required in order to determine the third mode frequency is greater than

that required to determine the first mode frequency, and while the first ten natural frequencies of one system may be obtained without difficulty, the significant figure capacity of the computer may be inadequate to determine the fourth mode frequency of another. Program VIPIPE is designed to terminate a solution when the number of significant figures required for its continuance exceeds that available in the machine.

The use of multiple precision arithmetic would extend VIPIPE'S utility considerably.

APPENDIX A

DESCRIPTION OF PROGRAM

A.1 General Remarks

Program VIPIPE is a CDC FORTRAN 60 [5] language digital computer program designed for use in the vibration analysis of planar piping systems. In and out-of-plane undamped natural frequencies may be found through its use. The analysis is performed by an application of the method of transfer matrices wherein each component of a system is characterized by a matrix called its transfer matrix. The entire system is characterized by the system transfer matrix obtained through an ordered multiplication of the component transfer matrices. Considerable flexibility with regard to the mathematical model employed to describe the system, and the nature of simplifying assumptions accepted in its vibration analysis, is provided in the program. For instance, although the method of solution yielding the greatest accuracy coupled with least machine time combines a distributed mass treatment of straight sections with a lumped mass treatment of curved sections, and considers the effects of both shear deflection and rotational inertia, the user may elect to use a lumped mass treatment of all sections and to consider or neglect, singly or together, the effects of shear deflection and rotational inertia.

Problem solution proceeds by an iterative process with radian frequency the independent variable. The sign of the frequency determinant[1] is used to control the search for and convergence to the system natural frequencies. The frequency increment is computed internally, as is the solution acceptability criterion. Both are accessible, however, and may be controlled by the user.[2] Subsectioning of components treated by lumped mass methods is carried out internally as a function of component L/D (length-to-diameter) ratio and the highest mode sought.

The program has the capability to analyze piping systems comprised of a maximum of fifty components, and having as many as twenty branches and twenty hangers.

A component, for program purposes, is defined as a curved section, i.e., a section of constant curvature, a straight section, a spring hanger, a flange or a coupling.[3] Although straight and curved sections, flanges and couplings must each be of constant outside diameter and wall thickness, these dimensions may vary from component to component. Similarly, intensive properties, i.e., shear and elastic moduli, and density, may not vary within a component, but may vary from component to component.

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1 See Chap. 2, 2.3 of the main body.
2 See App. B, section B.94.
3 See App. B, section B.95.

A shear area form factor of one half is built into the program. It may be altered at the user's discretion by altering one card.[1]

Five common end or boundary conditions, for the ends of the main member and the remote ends of any branches, may be specified by a simple coded input. Unusual boundary conditions may be considered by recourse to a more complicated input.[2]

Systems having rigid intermediate supports and/or non linear elements may not be analyzed by the use of this program. The number of natural frequencies found for a given system is a function of the significant figure capacity of the computer employed since the number of significant figures required in a solution is a function of the characteristics of the system under analysis, and for any system increases with increasing mode number.

A.2 Program Structure

The program consists of a six section main body and twenty-four subroutines. If sine, cosine, sinh, and cosh function routines are not available in the FORTRAN library they must be provided by other means since, although required by the program, they are not explicitly contained therein. The function of each section of the main body and of each subroutine is given below.

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[1] See App. B, section B.91.
[2] See App. B, section B.64.

A.21 Main Body Sections and their Functions

INPUT	Control read-in of data, allocate storage locations for arrays and subscripted variables.
INVARIANTS	Compute, and assign appropriate subscripts to, such system component characteristics as moment of inertia, and radius of gyration.
CONTROL 1	Construct transfer matrix of the system for the in-plane[1] case by calling for various subroutines in a sequence governed by the system geometry, the mathematical model specified, and the desired assumptions.
CONTROL 2	CONTROL 2 performs the same function as CONTROL 1 but for the out-of-plane[2] case.
ITERATION	Apply boundary conditions to the system transfer matrix constructed by CONTROL 1 or 2 so as to obtain the system frequency determinant. Evaluate the determinant and utilize its sign[3] in controlling the iterative search for mode frequencies. Initiate output when the specified frequency range has been fully explored, the required number of modes found, or the significant figure limit of the computer reached.
OUTPUT	Control output format.

A.22 Subroutines and their Functions

SUBSEC	Subdivide components to be treated by lumping mass on the basis of L/D ratio (or ratio of radius of curvature to diameter[4] and included angle of arc, as applicable.)
DISTM	Construct transfer matrix of straight sections having distributed mass for Case 1.

[1] Henceforth Case 1.
[2] Henceforth Case 2.
[3] See App. B, section B.94.
[4] Henceforth RHO/D where RHO is the radius of curvature of the component.

DISTMO	Construct transfer matrix of straight sections having distributed mass for Case 2.
SFIELD	Construct the transfer matrix of straight massless subsections for Case 1.
SFIELO	Construct the transfer matrix of straight massless subsections for Case 2.
CFIELD	Construct the transfer matrix of curved massless subsections for Case 1.
CFIELO	Construct the transfer matrix of massless curved subsections for Case 2.
POINT	Construct the transfer matrix of a point mass for Case 1.
POINO	Construct the transfer matrix of a point mass for Case 2.
MATMUL	Construct the transfer matrix of those components treated by lumping mass through the ordered matrix multiplication of the transfer matrices of massless subsections and point masses.
RIGID	Construct the transfer matrix of a rigid straight section having distributed mass for Case 1.
RIGIO	Construct the transfer matrix of a rigid straight section having distributed mass for Case 2.
STIFCO	Construct the transfer matrix of a massless rigid corner (essentially a transformation of coordinates at an elbow) for Case 1.
STIFOO	Construct the transfer matrix of a massless rigid corner (essentially a transformation of coordinates at an elbow) for Case 2.
HANGER	Construct the transfer matrix of a spring hanger for Case 1.
HANGEO	Construct the transfer matrix of a spring hanger for Case 2.
BRANCH	Construct a transfer matrix from the transfer matrix of a branch and its remote end

	boundary conditions, which permits a branch to be treated as a component of the main member, for Case 1.
BRANCO	Construct a transfer matrix from the transfer matrix of a branch and its remote end boundary conditions, which permits a branch to be treated as a component of the main member for Case 2.
STAVEC	Construct boundary condition vectors from coded input for Case 1.
STAVEO	Construct boundary condition vectors from coded input for Case 2.
INVERT	Invert a matrix.
FINMAT	Construct the transfer matrix of the system by successive premultiplication of the partial system transfer matrix by the transfer matrix of each system component as soon as each of the latter is constructed.
FINBRA	Construct the transfer matrix of a branch by successive premultiplication of the partial branch transfer matrix by the transfer matrix of each branch component as soon as each of the latter is constructed.
DELMA	Construct the frequency determinant of the system by applying the boundary conditions of the main member to the completed system transfer matrix.

A.3 Program Nomenclature

A.31 Main Body

A	Array/matrix. Transfer matrix of a massless subsection.
AA	Discriminant. A control in the convergence sequence.
AAMU	In INPUT; density (lbs./cu.ft.) of the first system component.

	In INVARIANTS; average mass per unit length of the main member.[1]
AMU	In INPUT; component density (lbs./cu. ft.). In remainder of program; component mass per unit length.
AD	Average outside diameter of the main member.[2]
ADOM	Current computed iteration frequency increment.
AFR	Average moment of inertia of the main member.[3]
AE	Elastic modulus of the first system component.
AG	Shear modulus of the first system component.
AID	Average inside diameter of the main member.[4]
AIX	Component radius of gyration about the x axis.
AIY	Component radius of gyration about the y or z axis.
AJ	Component moment of inertia about the y or z axis.
AJT	Component polar moment of inertia.
AL	Total length of main member.
AMYK	Discriminant. Controls choice of mathematical model, i.e., lumped or distributed mass.
B	Matrix. Transfer matrix of a point mass.
BB	Discriminant. Controls iteration frequency during convergence.

[1] Employed in computing initial frequency increment and solution acceptability criterion.

[2-4] Employed in computing the initial frequency increment and solution acceptability criterion.

BDOM	Initial frequency increment.
BANU	Component density (lbs./cu. ft.)
BM	Number of last mode found.
BR	Initial solution acceptability criterion.
CC	Discriminant. Designates final iteration during convergence.
CLX	Linear spring constant of a hanger in the x direction.[1]
CLY	Linear spring constant of a hanger in the y direction.[2]
CLZ	Linear spring constant of a hanger in the z direction.[3]
CR	Current value of the solution acceptability criterion.
CTX	Torsional spring constant of a hanger about the x axis.[4]
CTY	Torsional spring constant of a hanger about the y axis.[5]
CTZ	Torsional spring constant of a hanger about the z axis.[6]
D	Component outside diameter.
DD	Discriminant. Controls read-in and storage of the elastic and shear moduli and density of components.
DI	In INPUT; component wall thickness. in remainder of program; component inside diameter.
DT	Component wall thickness.
DOM	Current frequency increment during convergence.

1-6No coupling is permitted in action of hanger. For treatment of hangers in which coupling occurs see App. B, B.9.

E	Component elastic modulus.
EE	Average elastic modulus of the main member.
FFAC	Shear area form factor.
FR	First mode frequency of a fictitious straight beam having fixed ends, physical properties and transverse dimensions equal to an average of the characteristics of the main member to which it is equal in length.
G	Component shear modulus.
GG	Discriminant. Control in test for approach to significant figure limit of computer.
HDEL	Reference value of the frequency determinant (significant figure limit test).
HM	Number of highest mode sought.
I	Subscript.
II	Subscript which associates a branch with the angle at which it joins the main member.
IOP	Discriminant. Controls analysis to provide Case 1 and/or Case 2 mode frequencies as specified.
JK	Discriminant. Controls solution sequence during Case 1 or 2 analysis.
KK	Discriminant. Controls output to provide a print-out that an analysis has been terminated due to significant figure limitations when appropriate.
L	Iteration number.
LM	Number of current iteration.
M	Number of mode currently being sought.
MH	Number of highest mode found.
N	Subscript which associates a branch with its remote end boundary condition.

NBR	Number of branches in a system.
NM	Discriminant. Controls output.
NMBR	Total number of system boundary conditions.
NNBR	Discriminant. Assigns a component to the main member or to a branch, and if the latter, indicates its position in the branch.
NS	Total number of system components.
OMEGA	Current iteration frequency.
OMEGAC	Convergence reference frequency.
OMEGAG	Upper limit of frequency range of investigation.
OMEGAI	Investigation starting frequency.
OMEGAM	Mode frequency.
P	Sum of the numerical values of the spring constants of a component.
PDEL	Value of the frequency determinant used as a reference during iteration.
PHI	Angle at which a branch joins the main member.
R	Reciprocal of component flexural rigidity.
RHO	Component radius of curvature.
RI	Discriminant. Controls consideration of rotational inertia.
SD	Discriminant. Controls consideration of shear deflection.
SV	Coded boundary condition for Case 1.
SVIN	Non-coded boundary condition for Case 1.
SVO	Coded boundary condition for Case 2.
SVOP	Non-coded boundary condition for Case 2.
TDEL	Discriminant. Controls significant figure cut off.

THETA	Included angle of arc of curved components in degrees in input, radian measure in remainder of program.
THETR	Included angle of arc of curved components in degrees.
TL	Total length of a component.
U	Array name. Component transfer matrix.
UU	Array name. System transfer matrix.
V	Array name. System or branch transfer matrix.
VV	Array name. Branch transfer matrix.
Z	Number of subsections of a component.

A.32 Subroutines

a. SUBSEC

RHOD	Absolute value of radius of curvature of component.
RA	Component L/D ratio.
RD	Component RHO/D ratio.

b. DITM and DISTMO

ARA	Component cross sectional area.
AREA	Component equivalent shear area.
AL1 AL2 CBE CHL1 CL2 C00 C01 C02 C03 BE CL1 P4 S SBE SHL1 SL2 SPT W	Array elements and/or computed constants used in evaluating array elements.

RR	Component flexural rigidity.
W	Component torsional stiffness (DISTMO).

c. SFIELD and SFIELO

ARA	Component cross sectional area (SFIELD only).
AREA	Component equivalent shear area.
DL	Length of subsection.

d. CFIELD and CFIELO

ARA	Component cross sectional area.
AREA	Component equivalent shear area (CFIELD only).
CP, SP, F1, F3, F5, F6	Array elements and/or constants used in computing array elements.
W	Component shear stiffness (CFIELO only).
PHI	Included angle of arc of subsection.
RD	Absolute value of radius of curvature (CFIELD only).
W	Component longitudinal stiffness (CFIELD only).
RHOD	Signed component radius of curvature (CFIELO only).
SD	Discriminant. Controls consideration of shear deflection (CFIELO only).
V	Factor. Imparts sign to certain array elements.

e. POINT, POINO, PIGID, and PIGIO

AM	Total mass of subsection or component.

f. STIFCO and STIFOO

V	Factor. Imparts sign to certain array elements.

g. BRANCH and BRANCO

VV	Branch transfer matrix (BRANCH)
UV	Branch transfer matrix (BRANCO).
SVB	Array name. Remote end boundary condition vector (BRANCH).
SR	Array name. Remote end boundary condition vector (BRANCO).
R1, R2	Array names. Submatrices of branch transfer matrix.
R	Array name. Matrix product of R2 $\cdot$ R1^{-1} in first usage, spring matrix of branch in second usage in BRANCH.
S	Array name. Spring matrix of branch BRANCO.
G1, G2	Coordinate transformation matrix elements.
U	Array name. Transfer matrix which represents a branch as a component of the main member.

APPENDIX B

INSTRUCTIONS FOR PROGRAM USE

B.1 General Remarks

Appendix B is intended to provide complete instructions for using program VIPIPE. To that end, since for problem programming purposes, most system components must be identified with respect to a coordinate reference, a description of the reference system employed is given first. It is followed by a summary of data required, a description of the component identification system, an explanation of the nature of a state vector in terms of whose elements the system's boundary conditions, and its displacements and internal force systems are described, and a description of the data input format. The appendix concludes with some comments on the options available in the program.

For the purpose of setting up a problem for solution, the longest continuous run of pipe, or the one from which all branches emanate if these are not one and the same, must be designated as the main member. A system component is defined as a section of pipe, straight or of constant curvature, an elbow, a spring hanger, a coupling[1] or a flange. Diameter, wall thickness, elastic and shear moduli, and density may vary from component to component, but may not vary within a component. Since both torsional and linear spring rates (for hangers) appear internally as

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[1] See App. B, section B.95.

elements of a matrix used in obtaining matrix products, they must have finite values. Thus perfectly rigid hangers may not be treated exactly, but they can be approximated by using very large spring constants.

B.2 Coordinate System

In accordance with the procedures given below, a local coordinate system is constructed at each point of interest in the system. These points are usually the ends of each component, the beginning and end of the main member, and both ends of each branch. The coordinate systems are established with the piping system quiescent. Once established, they remain fixed in space.

In establishing a coordinate system, one end of the main member, henceforth called the left end, must be chosen as a starting point.

A right-handed coordinate system is used whose x axis, always directed away from the left end, is the local tangent to the central axis of the main member. The origin of the coordinate system lies on the main member central axis, and the xy plane coincides with the quiescent plane of the system. The z axis is always directed in the same direction, and this direction shall be called downward. The direction of the local y axis is chosen so as to complete the right-handed system.

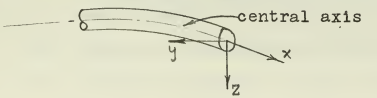

Fig. B.2 Coordinate System

The starting point, for reference purposes, of a branch is taken as its far end, i.e., the end farthest removed from the intersection of the branch with the main member. The plane of the branch coincides with the xy plane. The positive z axis of the branch and main member are parallel and have the same sense. The y axis is oriented so as to complete a right-handed system.

B.3 Branch Joining Angle

The angle at which a branch joins the main member is measured counter-clockwise in degrees from the x axis of the main member as it is oriented immediately before the intersection, to the x axis of the branch as it is oriented at the point of intersection.

B.4 Information Required to Program Problem

B.41 General

a. number of highest mode sought.

b. frequency range over which investigation is to be performed (initial and final frequencies in radians per second).

c. whether in-plane, out-of-plane or both in- and out-of-plane mode frequencies are sought.

d. whether employment of a distributed or of a lumped mass model is desired.

e. whether it is desired to consider or ignore singly or together the effects of shear deflection and rotational inertia.

f. boundary or support conditions at the ends of the main member and at the remote end of each branch.

g. number of components in the system.

h. number of branches in the system.

B.42 All Components Except Hangers

a. outside diameter (inches).

b. wall thickness (inches).

c. density (lbs./cu. ft).

d. shear modulus (psi).

e. elastic modulus (psi).

B.43 Straight Sections

a. length (inches).

B.44 Curved Sections

a. signed radius of curvature (negative if center of curvature lies on the negative y axis, positive if on the positive y axis).

b. arc central angle (degrees).

B.45 Hangers[1]

a. linear spring constant in x direction (lbs./in.).

b. linear spring constant in y direction (lbs./in.).

c. linear spring constant in z direction (lbs./in.).

d. torsional spring constant about x axis (in.-lbs./rad.).

e. torsional spring constant about y axis (in.-lbs./rad.).

f. torsional spring constant about z axis (in.-lbs./rad.).

B.5 Component Identification System

Components are identified by two numbers. The first, a component sequence number, is used to order data cards containing component extensive and intensive property data.

[1]See App. B., section B.96, Chap. 2, section 2.2a.

The second, a branch component identification number, is read in as data and serves to indicate whether or not a component is a member of a branch and if so, the position it occupies in the branch.

In addition, all branches, if any, are assigned a sequence number which is used to order data and/or data cards pertaining to branch boundary conditions and joining angles.

The methods of assigning these numbers are given below.

B.51 Component Sequence Number

All system components bear a component sequence number. The number one is assigned to the leftmost component of the main member. Each succeeding component encountered in tracing the main member is assigned a number one greater than that assigned to its lefthand neighbor. If and when a branch is encountered its components are also assigned sequence numbers beginning with the component farthest removed from intersection of the branch with the main member. That component is assigned a number one greater than that assigned to the last component of the main member encountered prior to, and abutting on, the junction. When more than one branch joins the main member at the same point, the user is free to choose the order in which desired to take up the branches for the purpose of assigning sequence numbers to their

components. Having established the order, the first branch is treated as if it were the only branch. The remaining branches are treated in exactly the same manner except that the component of the preceeding branch having the highest sequence number, i.e., the one abutting the joint, assumes the role performed by the last encountered main member component in treating the first branch (see Fig. B.51).

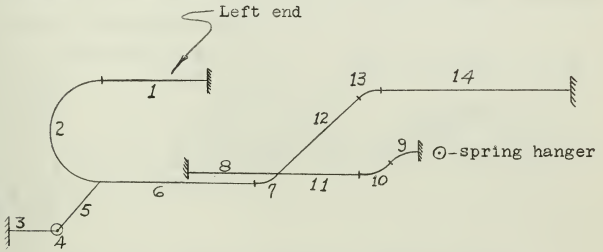

Fig. B51 Component Sequence Numbers

B.52 Branch Component Identification Number

Assignment of a component to a branch is indicated by the value of a fixed point constant (NNBR) coded as follows:

a. NNBR=1 if component is first member of a branch, i.e., the component farthest removed from the intersection.

b. NNBR=2 if component is neither the first nor the last member of the branch.

c. NNBR=3 if the component is the last member of the branch.

d. NNBR=-1 if the branch has only one component.

e. NNBR=0 all main member components.

B.53 Branch Sequence Number

Branches are numbered sequentially as they are encountered in tracing the main member. If two or more branches join the main member at the same point, the branch whose components have been assigned the lowest sequence numbers is assigned the lower or lowest branch sequence number (see Fig. B.53).

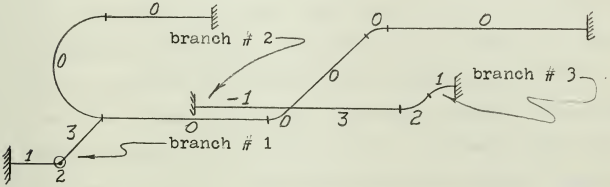

Fig. B.53 Branch component identification and branch sequence numbers.

B.6 Boundary Condition Codification

A boundary condition of the main member, or of a branch if any, consists of six parameters which, in the program, constitute the elements of a column matrix called a state vector. Five common boundary conditions, those of a fixed, free, pinned, propped, or a roller supported end may be indicated by a code given below which the program can interpret, and from which it constructs the required state vector.

B.61 Boundary Condition Code

Codable boundary conditions are indicated by the below listed values of the floating point constant SV:

a. SV=1. fixed end

b. SV=2. free end

c. SV=3. pinned end[1]

d. SV=4. propped end[2]

e. SV=5. roller supported end[3]

When a boundary condition is other than one of the five expressable by this code, the user must construct the necessary state vector and cause it to be read in as data. This situation will almost always obtain when both Case 1 and Case 2 mode frequencies are sought for the same system. It is apparent, for instance, that a constraint which fits the definition of a prop when considering Case 1 vibrations, may not be describable by any of the codes available when Case 2 vibrations are considered.

In constructing a state vector it is only necessary to know whether the value of a constituent parameter is zero or non-zero. The code for a parameter having a value of zero is 0., while that for one having a non-zero value is 1..

The constitution of the state vectors for Cases 1 and 2 are given below.

[1-3]See App. B, B.64 Catalog of Coded State Vectors

B.62 In-Plane State Vector (Case 1)

The vector elements in order are:

u displacement in the x direction.

v displacement in the y direction.

ϑ slope, in the xy plane, with respect to the x axis.

M_z moment about the z axis.

V_y shear in the y direction.

N normal force in the x direction.

B.63 Out-of-Plane State Vector (Case 2)

δ twist about the x axis.

T torque about the x axis.

-w (negative of) deflection in the z direction.

ψ slope, in the xz plane, with respect to the x axis.

M_y moment about the y axis.

V_z shear in the z direction.

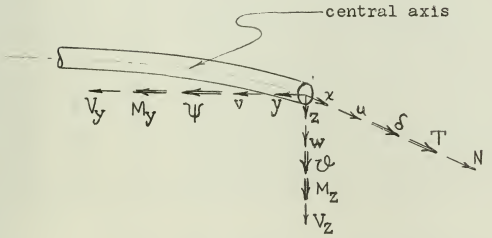

Fig. B.63 Vectorial representation of state vector parameters

B.64 Catalog of Coded State Vectors

a. Case 1.

State Vector Parameter	Boundary Conditions: fixed	free	pinned	propped	roller support
u	0.	1.	0.	1.	0.
v	0.	1.	0.	0.	1.
ϑ	0.	1.	1.	1.	1.
M_z	1.	0.	0.	0.	0.
V_y	1.	0.	1.	1.	0.
N	1.	0.	1.	0.	1.

b. Case 2.

State Vector Parameter	Boundary Conditions: fixed	free	pinned	propped	roller support
δ	0.	1.	0.	1.	1.
T	1.	0.	1.	0.	0.
-w	0.	1.	0.	0.	1.
ψ	0.	1.	1.	1.	1.
M_y	1.	0.	0.	0.	0.
V_z	1.	0.	1.	1.	0.

B.7 Format of Data

B.71 Problem Statement

The first data card contains a statement of the problem. The card has ten fields under the control of the following field specifications:

3I2 , 4F3.0 , 2F8.0 , F2.0

Field names and the information each field conveys are given below followed by an explanation of codes used.

Field	Name	and Content
1.	NS	number of components.
2.	NBR	number of branches.
3.	IOP	code calling for in and/or out-of-plane solution
4.	HM	number of modes sought.
5.	AMYK	code calling for the use of a lumped or distributed mass model in analysis.
6.	SD	code calling for inclusion or neglect of shear deflection effects.
7.	RI	code calling for inclusion or neglect of rotational inertia effects.
8.	OMEGAG	upper limit of frequency range of investigation (radians/sec.).
9.	OMEGAI	lower limit of frequency range of investigation and initial interation frequency. If a zero is placed in this field the search for mode frequencies will begin at .01 radians per second.
10.	GDABC	code calling for print out of the frequency and value of the frequency determinant for each iteration, the number of subsections assigned each component treated by lumping mass, the boundary condition state vector for both ends of the main member and of each branch, the number of components in the system, the frequency range of investigation, the highest mode sought, and all of the input data.

Codes used in the first data card are as follows:

a. IOP=0 both in-and out-of-plane mode frequencies sought.

b. IOP=1 in-plane frequencies sought.

c. IOP=2 out-of-plane mode frequencies sought.

d. AMYK=1 lumped mass model desired.

e. AMYK=0 distributed mass model desired.

f. SD=0 consider shear deflection.

g. SD=1 neglect shear deflection.

h. RI=0 consider rotational inertia.

i. RI=1 neglect rotational inertia.

j. GDABC=0 print solution only.

k. GDABC=1 print solution and input data.

l. GDABC=2 print solution, input data, and graph data.

B.72 Boundary Conditions

Boundary conditions for the in-and out-of-plane case are read in separately. Codes outlined in B.61 apply to both cases. When both in-and out-of-plane mode frequencies are sought, the boundary conditions for the in-plane cases are read in first.

The field specification for the first boundary condition data card is:

22F 3.0

The first N+2 fields of this card must be filled where N is the number of branches in the system. The coded representation of the main member's left and right end boundary

conditions are entered in the first and last fields used respectively. The codification of the far end boundary condition for each branch is entered in the intermediate fields in accordance with the branch sequence number (B.53). When a boundary condition is not describable by one of the codes given in B.61, a 0. must be entered in its data field. For each zero entered in the first boundary condition data card an additional data card is required on which is entered the actual state vector as given in B.62 or B.63, codified in accordance with the last paragraph of B.61. These cards[1] are placed immediately behind the first boundary condition data card their order dictated by the sequence number of the branch to which they apply (B.53).

B.73 Branch Joining Angle

When the system includes branches, the angles at which the branches join the main member are entered in one or two data cards as required whose field specification is:

20 F 7.3

For an N branch system the first N fields are used. The angles, in degrees, are entered in successive fields in accordance with the sequence number (B.53) of the branch to which they apply.

B.74 Component Extensive Properties and Branch Component Identification

A data card is required for each system component. The order of these cards is dictated by their component's

[1]Field specification 6F2.0.

sequence number (B.51). The card field specification is:

3F 5.2 , 2F 6.2 , 6F 7.2 , I2

Field names and their data content are given, in order, below:

D	outside diameter (inches)[1]
DI	wall thickness (inches).[2]
RHO	radius of curvature with sign (B.44) (inches). Note: Applies to curved components only. A zero (0.) is entered in all other cases.
TL	length (inches). Note: applies to straight components only. A zero (0.) is entered for all other cases.
THETA	are central angle (degrees). Note: Curved components only. A zero (0.) is entered in all other cases.
CLX, CLY, CLZ, CTX, CTY, CTZ	spring constants (B.45)[3]
NNBR	branch component identification number (B.42).

B.75 Intensive Properties of Components

The first intensive properties data card has four fields whose specifications are

F 7.3 , 2E 10.2 , F 2.0

[1-2] These fields are left blank in cards pertaining to spring hangers.

[3] These fields are left blank except in cards pertaining to spring hangers.

The field names and their data content are given below.

AAMU	density of first component (lbs./cu. ft.).
AE	elastic modulus of first component (psi).
AG	shear modulus of first component (psi).
DD	code. see * below.

* When the intensive properties of all components (spring hangers excluded) are identical, the DD field is left blank and the first intensive properties data card becomes the only intensive properties data card. If DD is non-zero, the data on the first card is interpreted as pertaining to the first component only and an intensive properties data card must be prepared for each of the remaining components, i.e., the second through the last, spring hangers included. These additional cards are identical to the first card in format and field content except that the DD field is deleted. A blank card must be inserted for each hanger. Intensive properties data cards are ordered in accordance with their component's sequence number (B.51).

B.8 Data Deck Assembly

The data deck is ordered as follows:

1. problem statement card.
2. boundary condition data card(s).
3. branch joining angle data card(s). (deleted when system has no branches).
4. component extensive properties data card(s).
5. component intensive properties data card(s).

The user may obtain solutions for any number of problems desired by "stacking" assembled problem data card decks at the end of the program deck. Note that a blank card must be inserted between the last card of the program deck and the problem statement card of the first problem. A blank card must also be placed behind the last intensive properties data card of the last problem. DO NOT insert blank cards between problems. The problem statement card of the N+first problem must be immediately behind the last intensive properties data card of the N^{th} problem.

B.9 Flexibilities of the Program VIPIPE

B.91 Shear Area Form Factor

A shear area form factor of one half is built into the program. [8] It may be altered by changing card 000580.

B.92 End Conditions

A properly constituted boundary condition state vector must have three zero and three non-zero elements. Frequently, however, an actual end condition may not meet this specification, e.g., an end of the main member or of a branch may be supported, or connected to a piece of machinery in such a manner as to approximate being held in a rubber mounting block (Fig. B.92-1). To program this

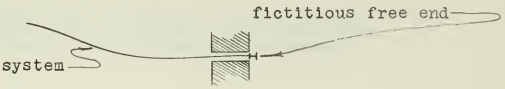

Fig. 92-1 Fictitious end condition

end condition, the user should make a good approximation of the elastic properties of the mounting, support, or connection in terms of the linear and torsional spring rates of the program (B.45, B.74)[2], then attach a fictitious, short (perhaps .01 in.) massless straight section to the end under consideration, and specify in the boundary condition for that end that the end is free.

Through the use of fictions of this sort, properly constituted end condition state vectors may be constructed for any real situation with little or no loss of accuracy.

B. 93 Subsectioning

Subsectioning for the lumped mass treatment of components is carried on in subroutine SUBSEC[1]. The subroutine is set up to divide a component into a maximum of ten subsections. The writer, through the analyses of several systems, found that this figure gave good accuracy of solution for a reasonable expenditure of machine time.

For all components having a length-to-diameter ratio greater than six, the number of subsections is computed as two times the value of a parameter HM which appears in the subroutine. The user may control its value by altering card 004720 in the subroutine.

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[1] See App. A, sections A.22, and A.32.
[2] i.e., treat the mount as a hanger.

B.94 Values of the Frequency Determinant and Solution Acceptability Criterion

The frequencies for which the value of the frequency determinant is zero are the natural frequencies of the system which it describes. In light of the number of significant figures available in a computer and of the effects of round-off, however, insistance that iteration and convergence operations continue until the value of the frequency determinant actually became zero would require an excessive expenditure of machine time. Instead, iteration proceeds until the sign of the value of the frequency determinant changes, then a convergence routine is carried out until the difference between the values of frequency, for which the corresponding value of the frequency determinant have opposite signs, is less than a specified value, CR. The result of a linear interpolation between those frequencies is accepted as a natural frequency of the system. The specified frequency difference CR is termed the solution acceptability criterion.

To compute a starting frequency increment, the program constructs a synthetic straight pipe, equal in length to the length of the main member, and having diameter, wall thickness, and intensive properties equal in magnitude to a weighted average of those properties of the composite main member. The fundamental frequency of the synthetic pipe is computed, and one half of its value is taken as the starting frequency increment.[1] The initial solution

[1] This increment may be changed by altering card 001090.

acceptability criterion is taken as one-twenty-fifth of the starting frequency increment. The initial solution acceptability criterion may be changed by altering card 001110.

The starting frequency increment and the initial solution acceptability criterion are used until the second natural frequency is found, after which the frequency increment is recomputed, as it is after each succeeding natural frequency is determined, and given a value equal to one half of the difference between the last two frequencies found. The new solution acceptability criterion is then recomputed as one-fiftieth of each new frequency increment. The new solution acceptability criterion may be changed by altering card 002800.

B.95 Flanges and Couplings

Pipe couplings and flanges represent real "lumps" of mass mounted in a piping system. They may be treated as individual system components. If this is desired, a component sequence number and a branch component identification number should be assigned in the usual manner (B.51, and B.52), and appropriate extensive and intensive properties data cards (B.74, and B.75) should be included in the data deck. These components are treated in the program, as rigid sections having distributed mass.

B.96 Hangers Exhibiting Coupling

Hangers such as that shown in Fig. B.96-1 can be treated with the tools available in VIPIPE by considering

the mounting arm as a branch having a hanger

Fig. B.96-1 Cantilevered hanger

and a free end. Recourse to similar methods will permit treatment of almost any hanger configuration.

B.97 Systems Having Unusual Flexibility

When the main member of a system is essentially cantilevered the computed starting frequency increment may be so large that the first two system natural frequencies are not found. For analysis of such systems it is recommended that the constant 4.73004019 in card 001080 be changed to 1.87510067. Similarly, if it is suspected that the fundamental frequency of the system is less than .01 rad./sec. a suitably lower starting frequency should be programed (B.71).

APPENDIX C

FLOW DIAGRAMS

C.1 General Remarks

Flow diagrams are included for all parts of the main body, and for several subroutines. Those subroutines for which no flow charts are included, with the exception of INVERT, incorporate, at most, two decisions in their structure. As applicable they sense on SD and/or RI (A.31) to change the value of one or two elements of the transfer matrix constructed by the particular subroutine.

Subroutine INVERT performs a matrix inversion by a pivotal technique. Sufficient comment statements are included in the subroutine to define its structure. If the user's FORTRAN library includes a satisfactory single precision matrix inversion subroutine, INVERT may be deleted after suitably modifying the dimension (cards 010070, 010080 BRANCH, and 011080, 011090 BRANCO) and call (card 010230 BRANCH, and 011660 BRANCO) statements of subroutines BRANCH and BRANCO.

C.2 Index of Flow Diagrams

C.21 Main Body

Flow Diagram	Page
INPUT	C-3
INVARIANTS	C-5
CONTROL IN-PLANE [1]	C-6
CONTROL OUT-OF-PLANE [1]	C-6
ITERATION	C-7
OUTPUT	C-9

[1] See C.3

C.22 Subroutines

Flow Diagram	Page
SUBSEC	C-10
MATMUL	C- 8
BRANCH	C-11
BRANCO	C-12
STAVEC	C-13
STAVEO	C-13
FINMAT	C-14
FINBRA	C-14
DELMA	C-14

C.3 Flow Diagrams[1] C-3 through C-14

Note: Since CONTROL IN-PLANE and CONTROL OUT-OF-PLANE are identical in structure, only the former flow diagram is included. They differ only in the subroutines called and the state vectors used (A.22). Subroutines SUBSEC, MATMUL, INVERT, FINMAT, FINBRA, and DELMA are common to both cases.

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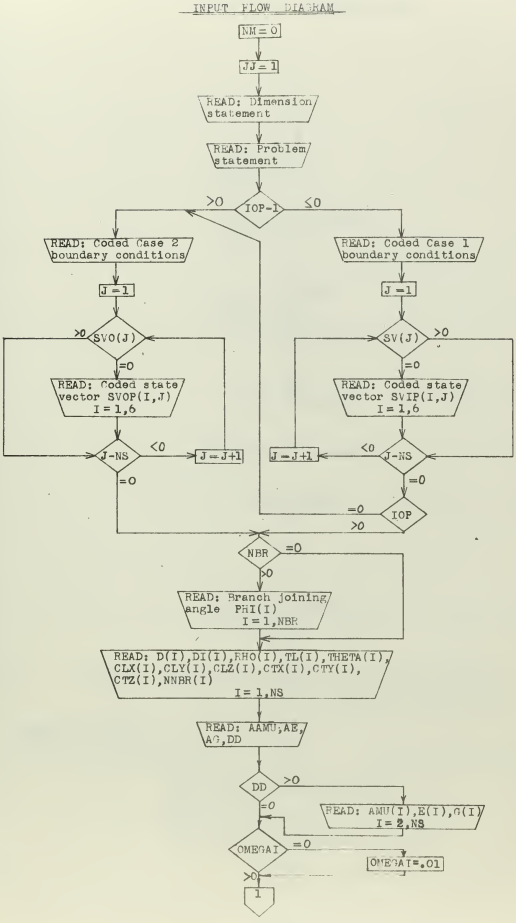
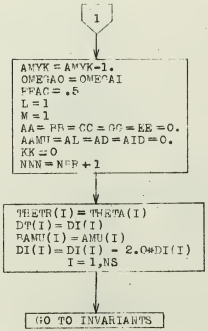
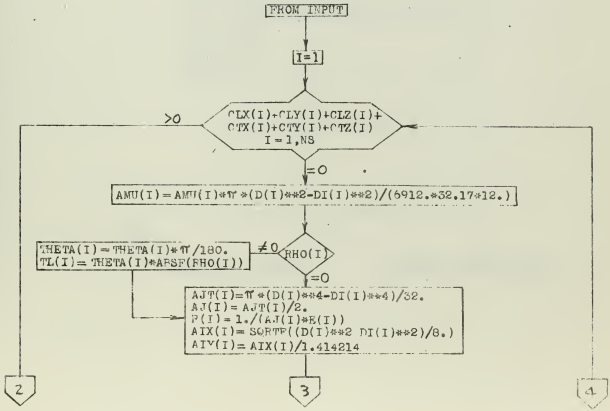
[1] A complete list of symbols is given in Appendix A, section A.3.

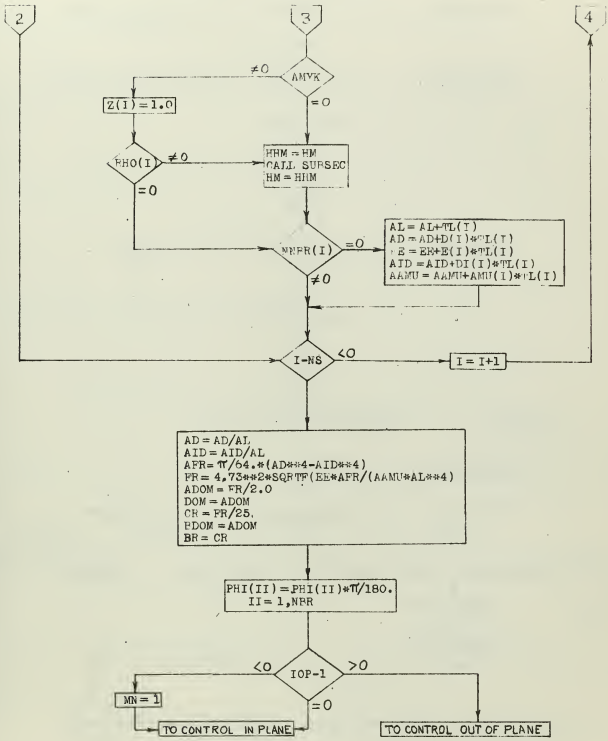

INPUT FLOW DIAGRAM
NM = 0
JJ = 1
READ: Dimension statement
READ: Problem statement
IOP-1
>0
≤0
READ: Coded Case 2 boundary conditions
READ: Coded Case 1 boundary conditions
J = 1
J = 1
SVO(J)
SV(J)
>0
=0
READ: Coded state vector SVOP(I,J) I = 1,6
READ: Coded state vector SVIP(I,J) I = 1,6
J-NS
<0
J = J+1
J = J+1
<0
J-NS
=0
=0
IOP
=0
>0
NBR
=0
>0
READ: Branch joining angle PHI(I) I = 1,NBR
READ: D(I),DI(I),RHO(I),TL(I),THETA(I), CLX(I),CLY(I),CLZ(I),CTX(I),CTY(I), CTZ(I),NNBR(I) I = 1,NS
READ: AAMU,AE, AG,DD
DD
>0
=0
READ: AMU(I),E(I),G(I) I = 2,NS
OMEGAI
=0
OMEGAI=.01
>0
1

INPUT FLOW DIAGRAM (CONT.)

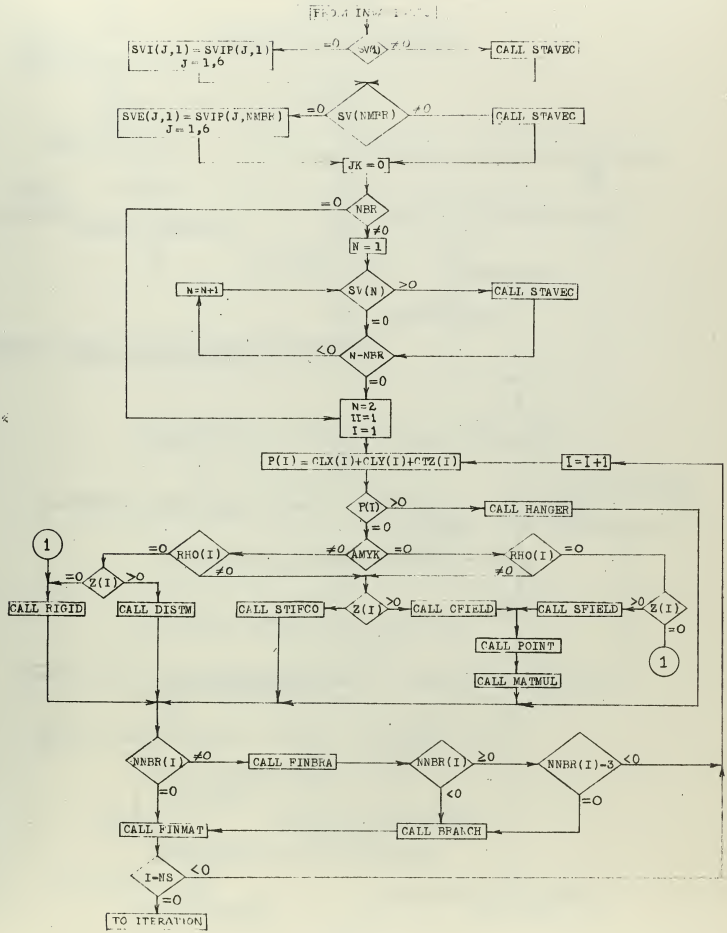

1
AMYK = AMYK-1.
OMETAO = OMEGAI
EPAC = .6
L = 1
M = 1
AA = BB = CC = GG = EE = 0.
AAMU = AL = AD = AID = 0.
KK = 0
NNN = NBR + 1
THETR(I) = THETA(I)
DT(I) = DI(I)
RAMU(I) = AMU(I)
DI(I) = DI(I) - 2.0*DI(I)
I = 1,NS
GO TO INVARIANTS

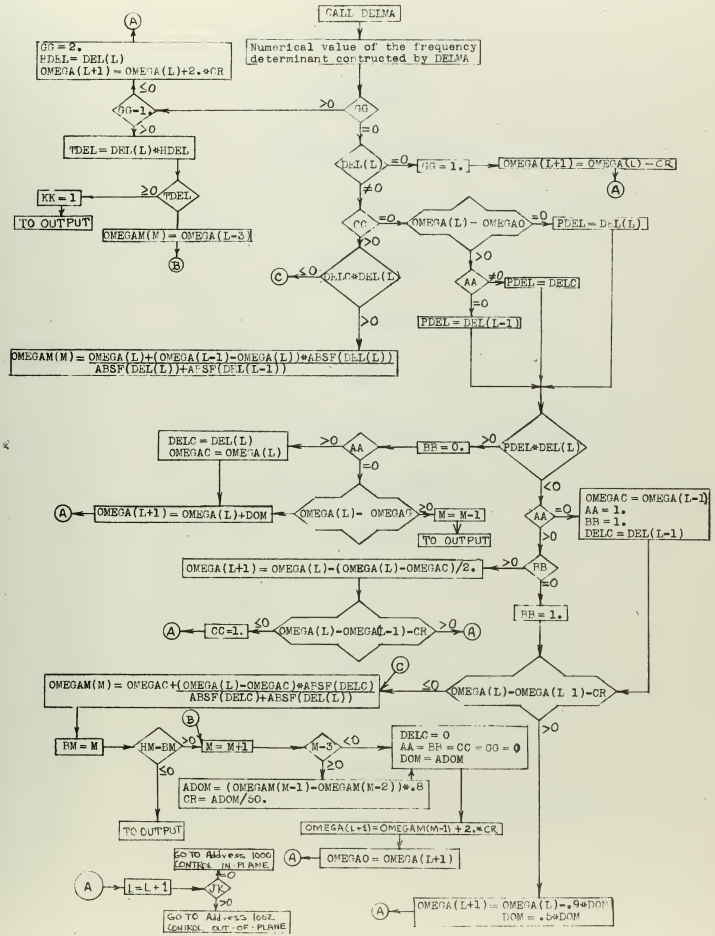
INVARIANTS FLOW DIAGRAM



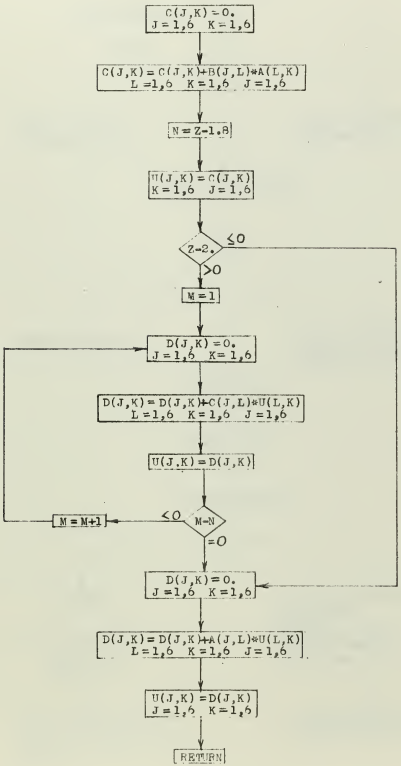

2
3
4
AMVK
≠0
=0
Z(I) = 1.0
PHO(I)
≠0
=0
HHM = HM
CALL SUBSEC
HM = HHM
NBFR(I)
=0
≠0
AL = AL+TL(I)
AD = AD+D(I)*TL(I)
E = ER+E(I)*TL(I)
AID = AID+DI(I)*TL(I)
AAMU = AAMU+AMU(I)*TL(I)
I-NS
<0
I = I+1
AD = AD/AL
AID = AID/AL
AFR= π/64.*(AD**4-AID**4)
FR = 4.73**2*SQRT(ER*AFR/(AAMU*AL**4)
ADOM = FR/2.0
DOM = ADOM
CR = FR/25.
FDOM = ADOM
BR = CR
PHI(II) = PHI(II)*π/180.
II = 1,NBR
IOP-1
<0
>0
=0
MN = 1
TO CONTROL IN PLANE
TO CONTROL OUT OF PLANE

CONTROL IN PLANE FLOW DIAGRAM

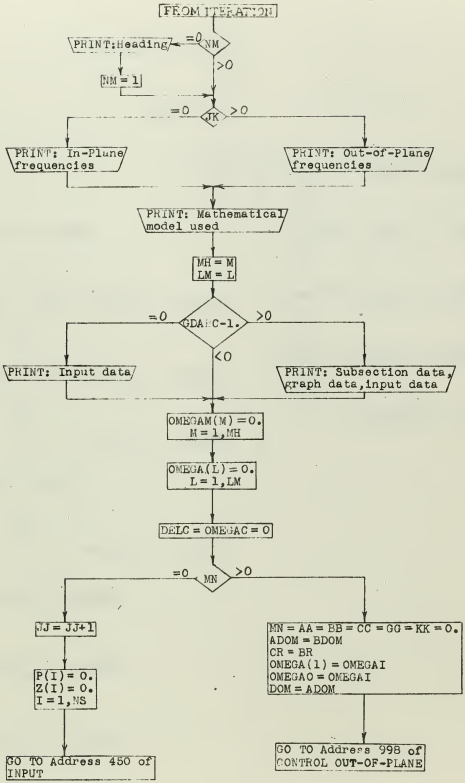


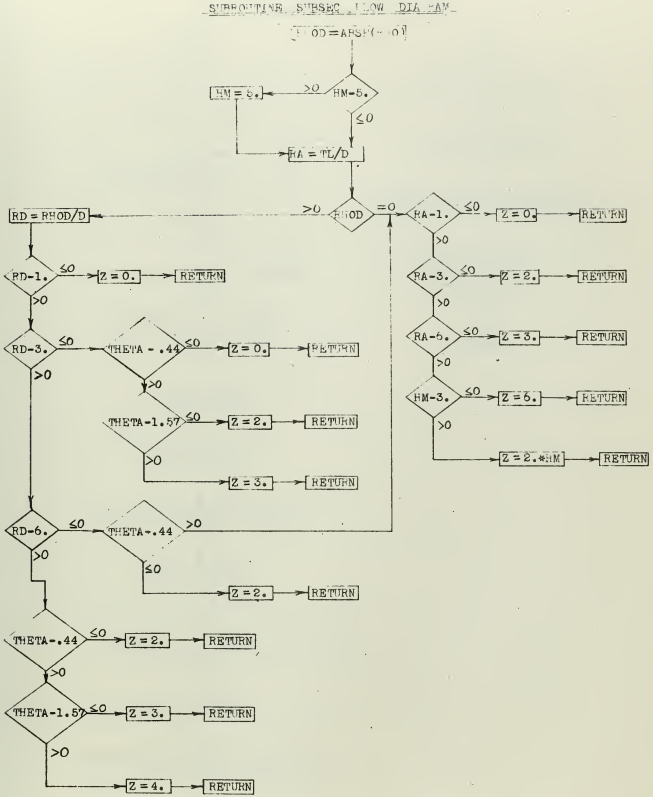

ITERATION FLOW DIAGRAM
CALL DELMA
Numerical value of the frequency determinant contructed by DELMA
GG = 2.
HDEL = DEL(L)
OMEGA(L+1) = OMEGA(L)+2.*CR
GG-1
GG
TDEL = DEL(L)*HDEL
DEL(L)
GG = 1.
OMEGA(L+1) = OMEGA(L) - CR
KK = 1
TDEL
TO OUTPUT
OMEGAM(M) = OMEGA(L-3)
CC
OMEGA(L) - OMEGAO
PDEL = DEL(L)
DELC*DEL(L)
AA
PDEL = DELC
PDEL = DEL(L-1)
OMEGAM(M) = OMEGA(L)+(OMEGA(L-1)-OMEGA(L))*ABSF(DEL(L))/(ABSF(DEL(L))+ABSF(DEL(L-1)))
DELC = DEL(L)
OMEGAC = OMEGA(L)
BB = 0.
PDEL*DEL(L)
OMEGA(L+1) = OMEGA(L)+DOM
OMEGA(L)- OMEGAO
M = M-1
TO OUTPUT
OMEGAC = OMEGA(L-1)
AA = 1.
BB = 1.
DELC = DEL(L-1)
OMEGA(L+1) = OMEGA(L)-(OMEGA(L)-OMEGAC)/2.
BB
BB = 1.
CC=1.
OMEGA(L)-OMEGAC(L-1)-CR
OMEGAM(M) = OMEGAC+(OMEGA(L)-OMEGAC)*ABSF(DELC)/(ABSF(DELC)+ABSF(DEL(L)))
OMEGA(L)-OMEGA(L-1)-CR
BM = M
HM-BM
M = M+1
M-3
DELC = 0
AA = BB = CC = GG = 0
DOM = ADOM
ADOM = (OMEGAM(M-1)-OMEGAM(M-2))*.8
CR = ADOM/50.
TO OUTPUT
OMEGA(L+1)=OMEGAM(M-1)+2.*CR
GO TO Address 1000 CONTROL IN PLANE
OMEGAO = OMEGA(L+1)
L=L+1
J/K
GO TO Address 1007 CONTROL OUT OF PLANE
OMEGA(L+1) = OMEGA(L)-.9*DOM
DOM = .9*DOM

SUBROUTINE MATMUL FLOW DIAGRAM

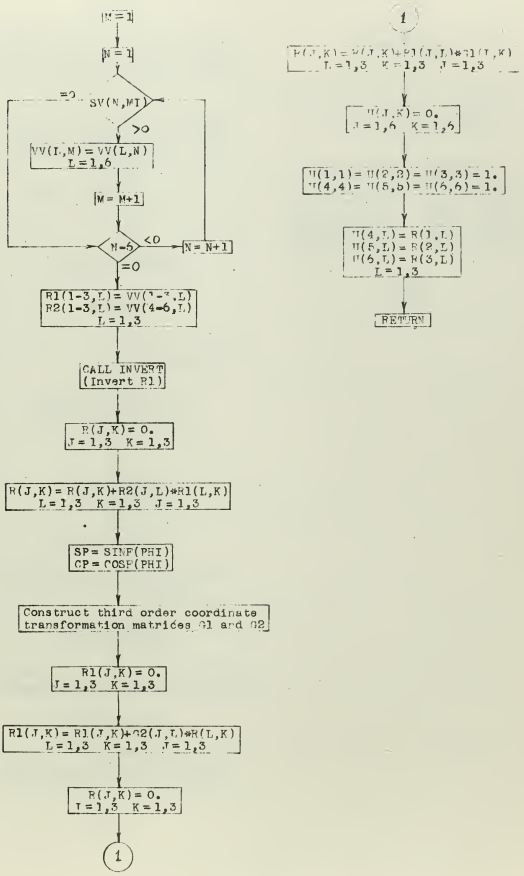


OUTPUT FLOW DIAGRAM

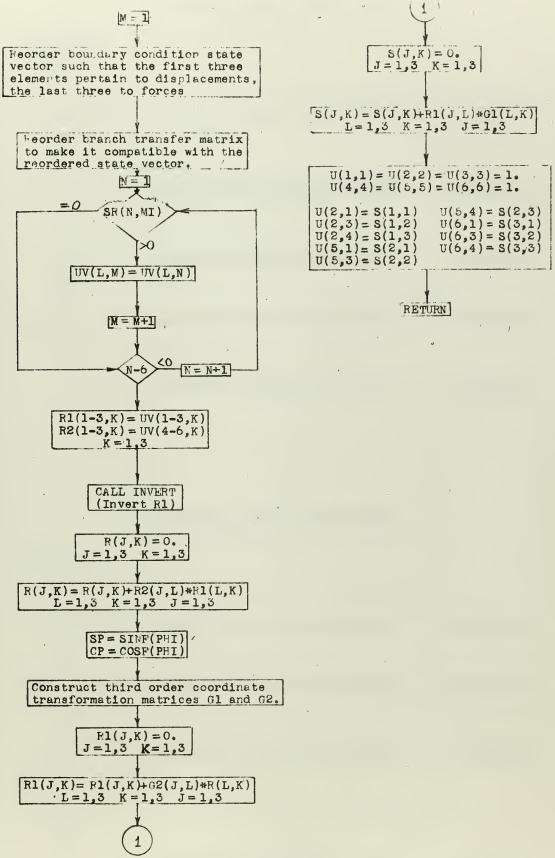



SUBROUTINE SUBSEC FLOW DIAGRAM
ROOD=ABS(-.01)
HM=5.
>0
HM-5.
≤0
RA=TL/D
RMOD
>0
=0
RD=RMOD/D
RA-1.
≤0
Z=0.
RETURN
>0
RD-1.
≤0
Z=0.
RETURN
>0
RA-3.
≤0
Z=2.
RETURN
>0
RD-3.
≤0
THETA-.44
≤0
Z=0.
RETURN
>0
RA-5.
≤0
Z=3.
RETURN
>0
THETA-1.57
≤0
Z=2.
RETURN
>0
HM-3.
≤0
Z=5.
RETURN
>0
Z=3.
RETURN
Z=2.*HM
RETURN
RD-6.
≤0
THETA-.44
>0
>0
≤0
Z=2.
RETURN
THETA-.44
≤0
Z=2.
RETURN
>0
THETA-1.57
≤0
Z=3.
RETURN
>0
Z=4.
RETURN

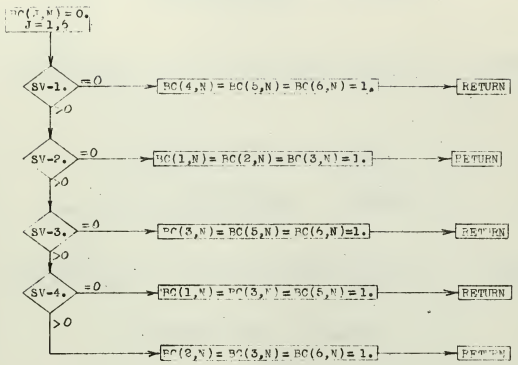
SUBROUTINE E. BRANCH. FLOW DIAGRAM



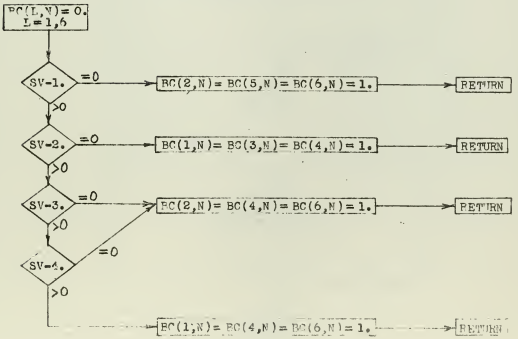
SUBROUTINE FRANCO FLOW DIAGRAM


M = 1
Reorder boundary condition state vector such that the first three elements pertain to displacements, the last three to forces
Reorder branch transfer matrix to make it compatible with the reordered state vector.
N = 1
SR(N,MI)
=0
>0
UV(L,M) = UV(L,N)
M = M+1
N-6
<0
N = N+1
R1(1-3,K) = UV(1-3,K)
R2(1-3,K) = UV(4-6,K)
K = 1,3
CALL INVERT (Invert R1)
R(J,K) = 0.
J = 1,3 K = 1,3
R(J,K) = R(J,K)+R2(J,L)*R1(L,K)
L = 1,3 K = 1,3 J = 1,3
SP = SINF(PHI)
CP = COSP(PHI)
Construct third order coordinate transformation matrices G1 and G2.
R1(J,K) = 0.
J = 1,3 K = 1,3
R1(J,K) = R1(J,K)+G2(J,L)*R(L,K)
L = 1,3 K = 1,3 J = 1,3
1
1
S(J,K) = 0.
J = 1,3 K = 1,3
S(J,K) = S(J,K)+R1(J,L)*G1(L,K)
L = 1,3 K = 1,3 J = 1,3
U(1,1) = U(2,2) = U(3,3) = 1.
U(4,4) = U(5,5) = U(6,6) = 1.
U(2,1) = S(1,1) U(5,4) = S(2,3)
U(2,3) = S(1,2) U(6,1) = S(3,1)
U(2,4) = S(1,3) U(6,3) = S(3,2)
U(5,1) = S(2,1) U(6,4) = S(3,3)
U(5,3) = S(2,2)
RETURN

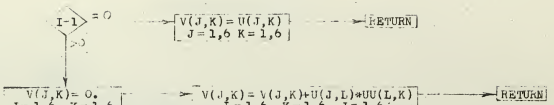
SUBROUTINE STAVEC FLOW DIAGRAM



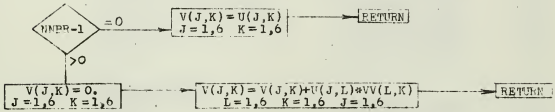
SUBROUTINE STAVEO FLOW DIAGRAM



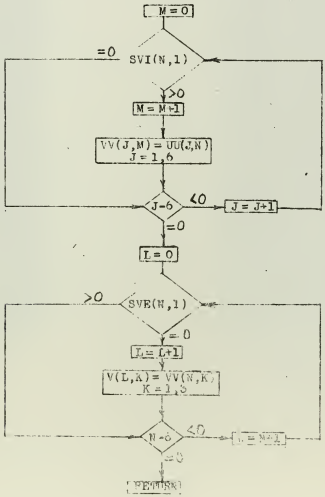
SUBROUTINE FINMAT FLOW DIAGRAM



SUBROUTINE FINEA FLOW DIAGRAM



SUBROUTINE DELMA FLOW DIAGRAM



APPENDIX D

PROGRAM LISTING

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D.11 Main Body

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ITERATION	002470-003260	D-6
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SUBSEC	004690-005140	D-10
DISTM	005170-005710	D-11
DISTMO	005740-006280	D-12
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CFIELD	006940-007370	D-14
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D.2 Listing D-2 through D-27

```
      PROGRAM VIPIPE
C                                                                       000020
C                                                                       000030
C     INPUT                                                             000040
C                                                                       000050
C                                                                       000060
      NM=0                                                              000070
      JJ=1                                                              000080
  4500DIMENSION AJI(50),AJ(50),R(50),AIX(50),AIY(50),PHI(20),SVI(6,1),  000090
     1SVB(6),D(50),RI(50),TL(50),Z(50),G(50),E(50),AMU(50),THETA(50),RHO 000100
     2(50),CMEGAM(20),EEL(500),CMEGA(500),A(6,6),B(6,6),U(6,6),CLX(50),C 000110
     3LY(50),CLZ(50),CTX(50),CTY(50),CTZ(50),V(6,6),VV(6,6),UU(6,6),SVE( 000120
     46,1),SV(22),SVIP(6,22),SVD(22),SVDP(6,22),NNBR(50),BAMU(50),THETR( 000130
     520),DT(50),P(20)                                                  000140
      READ 1500,NS,NBR,IOP,HH,AMYK,SD,RI,OMECAG,OMEGAI,SDABC            000150
 1500 FORMAT(3I2,4F3.0,2F8.0,F2.0)                                      000160
      NMBR=NBR+2                                                        000170
      IF(IOP-1)1501,1501,1506                                           000180
 1501 READ 1502,(SV(I),I=1,NMBR)                                        000190
 1502 FORMAT(22F3.0)                                                    000200
      DO 1505 J=1,NMBR                                                  000210
      IF(SV(J))1503,1503,1505                                           000220
 1503 READ 1504,(SVIP(I,J),I=1,6)                                       000230
 1504 FORMAT(6F2.0)                                                     000240
 1505 CONTINUE                                                          000250
      IF(IOP)1506,1506,1510                                             000260
 1506 READ 1507,(SVC(I),I=1,NMBR)                                       000270
 1507 FORMAT(22F3.0)                                                    000280
      DO 1510 J=1,NMBR                                                  000290
      IF(SVC(J))1508,1508,1510                                          000300
 1508 READ 1509 (SVDP(I,J),I=1,6)                                       000310
 1509 FORMAT(6F2.0)                                                     000320
 1510 CONTINUE                                                          000330
      IF(NBR)1513,1513,1511                                             000340
 1511 READ 1512,(PHI(I),I=1,NBR)                                        000350
 1512 FORMAT (10F7.3)                                                   000360
 1513DREAD 1514,(DI(I),DI(I),RHO(I),TL(I),THETA(I),CLX(I),CLY(I),CLZ(I),C 000370
     1TX(I),CTY(I),CTZ(I),NNBR(I),I=1,NS)                               000380
 1514 FORMAT(3F5.2,2F6.2,6F7.2,I2)                                      000390
      READ 1515 , AAMU,AE,AG,DE                                         000400
 1515 FORMAT (F7.3,2E10.2,F2.0)                                         000410
      IF(DE)1516,1516,1518                                              000420
 1516 DO 1517 I=1,NS                                                    000430
      AMU(I)=AAMU                                                       000440
      E(I)=AE                                                           000450
 1517 G(I)=AG                                                           000460
      GO TO 1520                                                        000470
 1518 AMU(1)=AAMU                                                       000480
      E(1)=AE                                                           000490
      G(1)=AG                                                           000500
      READ 1519,(AMU(I),E(I),G(I),I=2,NS)                               000510
 1519 FORMAT (F7.3,2E10.2)                                              000520
      IF(OMEGAI)1520,1520,1521                                          000530
 1520 OMEGAI=.01                                                        000540
 1521 OMEGA(1)=OMEGAI                                                   000550
```

```
      AMYK=AMYK-1.0                                                    000560
      OMEGAC=OMEGAI                                                    000570
      FFAC=.5                                                          000580
      L=1                                                              000590
      M=1                                                              000600
      AA=0.0                                                           000610
      BB=0.0                                                           000620
      CC=0.0                                                           000630
      GG=0.0                                                           000640
      KK=0                                                             000650
      AAMU=0.0                                                         000660
      EE=0.0                                                           000670
      AL=0.0                                                           000680
      AD=0.0                                                           000690
      AID=0.0                                                          000700
      NNN=NBR+1                                                        000710
      DO 1522 I=1,NS                                                   000720
      DT(I)=DI(I)                                                      000730
      BAMU(I)=AMU(I)                                                   000740
      THETR(I)=THETA(I)                                                000750
 1522 DI(I)=D(I)-2.0*DI(I)                                             000760
C                                                                      000770
C                                                                      000780
C     INVARIANTS                                                       000790
C                                                                      000800
      DO 100 I=1,NS                                                    000810
      IF(CLX(I)+CLY(I)+CLZ(I)+CTX(I)+CTY(I)+CTZ(I))100,101,100         000820
  101 AMU(I)=AMU(I)*3.14159265*(D(I)**2-DI(I)**2)/(6912.*32.17*12.)    000830
      IF(RHO(I))102,103,102                                            000840
  102 THETA(I)=THETA(I)*3.14159265/180.                                000850
      TL(I)=THETA(I)*ABSF(RHO(I))                                      000860
  103 AJT(I)=3.14159265*(D(I)**4-DI(I)**4)/32.                         000870
      AJ(I)=AJT(I)/2.                                                  000880
      R(I)=1.0/(AJ(I)*L(I))                                            000890
      AIX(I)=SQRTF((D(I)**2+DI(I)**2)/8.)                              000900
      AIY(I)=AIX(I)/1.414214                                           000910
      IF(AMYK)104,105,104                                              000920
  104 Z(I)=1.0                                                         000930
      IF(RHO(I))105,106,105                                            000940
  105 HHM=HM                                                           000950
      CALL SUBSEC (RHO(I),THETA(I),D(I),TL(I),HM,Z(I))                 000960
      HM=HHM                                                           000970
  106 IF(NNBR(I))108,107,108                                           000980
  107 AL=AL+TL(I)                                                      000990
      AD=AD+D(I)*TL(I)                                                 001000
      EE=EE+E(I)*TL(I)                                                 001010
      AID=AID+DI(I)*TL(I)                                              001020
      AAMU=AAMU+AMU(I)*TL(I)                                           001030
  108 CONTINUE                                                         001040
      AD=AD/AL                                                         001050
      AID=AID/AL                                                       001060
      AFR=3.14159265/64.*(AD**4-AID**4)                                001070
      FR=4.73004019**2*SQRTF(EE*AFR/(AAMU*AL**4))                      001080
      ADDM=FR/2.0                                                      001090
```

```
      NDM=ADDR                                                          0011CO
      C9=F9/25.0                                                        0011D0
      BDDM=ADDM                                                         0011E0
      BR=LR                                                             0011F0
      DO 109 II=1,NBR                                                   0012G0
  109 PHI(II)=PHI(II)*3.1415926535/180.0                                0012H0
      IF(ICP-1)110,999,998                                              0012I0
  110 MN=1                                                              0012J0
C                                                                       0012K0
C                                                                       0012L0
C     CONTROL  IN PLANE                                                 0012M0
C                                                                       0012N0
  999 IF(SV(1))1011,1021,1011                                           0012O0
 1011 CALL STAVEC (SV(1),1,SVI)                                         0012P0
      GO TO 1041                                                        0012Q0
 1021 DO 1031 J=1,6                                                     0012R0
 1031 SVI(J,1)=SVIP(J,1)                                                0012S0
 1041 IF(SV(NMBR))1051,1061,1051                                        0012T0
 1051 CALL STAVEC(SV(NMBR),1,SVE)                                       0012U0
      GO TO 1081                                                        0012V0
 1061 DO 1071 J=1,6                                                     0012W0
 1071 SVE(J,1)=SVIP(J,NMBR)                                             0012X0
 1081 JK=0                                                              0012Y0
      IF(NBR)1000,1000,1085                                             0012Z0
 1085 DO 1087 N=2,NNN                                                   0013A0
      IF(SV(N))1087,1087,1086                                           0013B0
 1086 CALL STAVEC(SV(N),N,SVIP)                                         0013C0
 1087 CONTINUE                                                          0013D0
 1000 N=2                                                               0013E0
      II=1                                                              0013F0
      DO 1001 I=1,NS                                                    0013G0
      P(I)=CLX(I)+CLY(I)+CTZ(I)                                         0013H0
      IF(P(I))37,38,37                                                  0013I0
   37 CALL HANGER(CLX(I),CLY(I),CTZ(I),U)                               0013J0
      GO TO 62                                                          0013K0
   38 IF(AMYK)39,41,39                                                  0013L0
   39 IF(RHO(I))40,42,40                                                0013M0
   40 IF(Z(I))43,45,43                                                  0013N0
   41 IF(RHO(I))40,48,40                                                0013O0
   42 IF(Z(I))46,47,46                                                  0013P0
   43 CALL GFIELD (R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),E(I),A) 0013Q0
   44 CALL POINT (AMU(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)               0013R0
      CALL MATMUL (A,B,Z(I),U)                                          0013S0
      GO TO 62                                                          0013T0
   45 CALL STIFCO (THETA(I),RHO(I),U)                                   0013U0
      GO TO 62                                                          0013V0
  460CALL DISTM (AMU(I),AIY(I),OMEGA(L),TL(I),R(I),D(I),DI(I),G(I),SD,R 0013W0
     1I,E(I),FFAC,U)                                                    0013X0
      GO TO 62                                                          0013Y0
   47 CALL RIGID (AMU(I),TL(I),OMEGA(L),AIY(I),RI,L)                    0013Z0
      GO TO 62                                                          0014A0
   48 IF(Z(I))49,47,49                                                  0014B0
   49 CALL SFIELD (DI(I),TL(I),R(I),Z(I),G(I),SD,D(I),E(I),FFAC,A)      0014C0
      GO TO 44                                                          0014D0
```

```
   62 IF(NNBR(I))65,63,65                                              001640
   63 CALL FINMAT (L,UU,I,V)                                           001650
      DO 64 J=1,6                                                      001660
      DO 64 K=1,6                                                      001670
   64 UU(J,K)=V(J,K)                                                   001680
      GO TO 1001                                                       001690
   65 CALL FINBRA (L,VV,NNBR(I),V)                                     001700
      DO 66 J=1,6                                                      001710
      DO 66 K=1,6                                                      001720
   66 VV(J,K)=V(J,K)                                                   001730
      IF(NNBR(I))68,67,67                                              001740
   67 IF(NNBR(I)-3)1001,68,68                                          001750
   68 CALL BRANCH(SVIP,VV,PHI(II),N,U)                                 001760
      N=N+1                                                            001770
      II=II+1                                                          001780
      GO TO 63                                                         001790
 1001 CONTINUE                                                         001800
      GO TO 2000                                                       001810
C                                                                      001820
C                                                                      001830
C     CONTROL  CUT OF PLANE                                            001840
C                                                                      001850
  998 IF(SVO(1))1013,1023,1013                                         001860
 1013 CALL STAVEO(SVO(1),1,SVI)                                        001870
      GO TO 1043                                                       001880
 1023 DO 1033 J=1,6                                                    001890
 1033 SVI(J,1)=SVCP(J,1)                                               001900
 1043 IF(SVO(NMBR))1053,1063,1053                                      001910
 1053 CALL STAVEO(SVO(NMBR),1,SVE)                                     001920
      GO TO 1083                                                       001930
 1063 DO 1073 J=1,6                                                    001940
 1073 SVE(J,1)=SVCP(J,NMBR)                                            001950
 1083 JK=1                                                             001960
      IF(NER)1002,1002,1093                                            001970
 1093 DO 1095 N=2,NAN                                                  001980
      IF(SVO(N))1095,1095,1094                                         001990
 1094 CALL STAVEO(SVO(N),N,SVCP)                                       002000
 1095 CONTINUE                                                         002010
 1002 N=2                                                              002020
      II=1                                                             002030
      DO 1003 I=1,NS                                                   002040
      P(I)=CTX(I)+CLZ(I)+CTY(I)                                        002050
      IF(P(I))1,2,1                                                    002060
    1 CALL HANGEO(CTX(I),CLZ(I),CTY(I),U)                              002070
      GO TO 14                                                         002080
    2 IF(AMYK)3,11,3                                                   002090
    3 IF(RHO(I))4,9,4                                                  002100
    4 IF(Z(I))6,5,6                                                    002110
    5 CALL STIFCO (THETA(I),RHO(I),U)                                  002120
      GO TO 14                                                         002130
   60CALL CFIELO(AJT(I),R(I),Z(I),G(I),SO,RHO(I),THETA(I),O(I),OI(I),FF 002140
     1AC,A)                                                            002150
    7 CALL PCINC (AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)       002160
```

```
      GO TO 14                                                          002180
    8 IF(Z(I))10,9,10                                                   002190
    9 CALL RIGID (AMU(I),TL(I),AIX(I),OMEGA(L),AIY(I),AJT(I),R(I),U)    002200
      GO TO 14                                                          002210
  100CALL DISTRD(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),AJT(I),R(I),U(I),D 002220
     1I(I),G(I),SG,RI,FFAC,W)                                           002230
      GO TO 14                                                          002240
   11 IF(RFC(I))4,12,4                                                  002250
   12 IF(Z(I))13,9,13                                                   002260
   13 CALL SFIELD (DI(I),TL(I),AJT(I),R(I),Z(I),G(I),SG,DI(I),FFAC,A)   002270
      GO TO 7                                                           002280
   14 IF(NNBR(I))17,15,17                                               002290
   15 CALL FINMAT (U,UU,I,V)                                            002300
      DO 16 J=1,6                                                       002310
      DO 16 K=1,6                                                       002320
   16 UU(J,K)=V(J,K)                                                    002330
      GO TO 1003                                                        002340
   17 CALL FINBRA (L,VV,NNBR(I),V)                                      002350
      DO 18 J=1,6                                                       002360
      DO 18 K=1,6                                                       002370
   18 VV(J,K)=V(J,K)                                                    002380
      IF(NNBR(I))82,19,19                                               002390
   19 IF(NNBR(I)-3)1003,82,82                                           002400
   82 CALL BRANCD(SVCP,VV,PHI(II),N,U)                                  002410
      N=N+1                                                             002420
      II=II+1                                                           002430
      GO TO 15                                                          002440
 1003 CONTINUE                                                          002450
C                                                                       002460
C       ITERATION                                                       002470
C                                                                       002480
 2000 CALL DELMA(UU,SVI,SVE,V)                                          002490
     1DEL(L)=V(1,1)*V(2,2)*V(3,3)-V(1,3)*V(2,2)*V(3,1)+V(1,2)*V(2,3)*V(3 002500
     1,1)-V(1,2)*V(2,1)*V(3,3)+V(1,3)*V(3,2)*V(2,1)-V(1,1)*V(3,2)*V(2,3) 002510
      GO TO 61                                                          002520
   21 PDEL=DEL(L)                                                       002530
      GO TO 25                                                          002540
   22 IF(AA)23,23,24                                                    002550
   23 PDEL=DEL(L-1)                                                     002560
      GO TO 25                                                          002570
   24 PDEL=DELC                                                         002580
   25 IF(PDEL*DEL(L))26,26,27                                           002590
   26 IF(AA)28,28,57                                                    002600
   27 BB=0                                                              002610
      IF(AA)35,35,52                                                    002620
   28 AA=1.0                                                            002630
      BB=1.0                                                            002640
      DELC=DEL(L-1)                                                     002650
      CMEGAC=CMEGA(L-1)                                                 002660
      GO TO 59                                                          002670
 2900MEGAM(M)=OMEGAC+(OMEGA(L)-CMEGAC)*ABSF(DELC)/(ABSF(DELC)+          002680
     1ABSF(DEL(L)))                                                     002690
      GO TO 32                                                          002700
   30 OMEGA(L+1)=OMEGA(L)-.9*DCM                                        002710
```

```
      DDM=.5*DDM                                                        002720
   31 L=L+1                                                             002730
      IF(JK)1000,1000,1002                                              002740
   32 BM=M                                                              002750
      IF(BM-BM)3000,3000,33                                             002760
   33 M=M+1                                                             002770
      IF(M-3)340,34,34                                                  002780
   34 ADDM=(OMEGAM(M-1)-OMEGAM(M-2))*.5                                 002790
      CR=ADDM/50.                                                       002800
  340 AA=0.0                                                            002810
      BB=0                                                              002820
      CC=0                                                              002830
      GG=0.0                                                            002840
      DDM=ADDM                                                          002850
      DELC=0                                                            002860
      OMEGA(L+1)=OMEGAM(M-1)+2.*CR                                      002870
      OMEGAC=OMEGA(L+1)                                                 002880
      GO TO 31                                                          002890
   35 IF(OMEGA(L)-OMEGAC)53,60,60                                       002900
   50 OMEGA(L+1)=OMEGA(L)-(OMEGA(L)-OMEGAC)/2.0                         002910
      IF(OMEGA(L)-OMEGA(L+1)-CR)51,51,31                                002920
   51 CC=1.0                                                            002930
      GO TO 31                                                          002940
   52 DELC=DEL(L)                                                       002950
      OMEGAC=OMEGA(L)                                                   002960
   53 OMEGA(L+1)=OMEGA(L)+DDM                                           002970
      GO TO 31                                                          002980
   54 IF(DELC*DEL(L))29,29,55                                           002990
  550MEGAM(M)=OMEGA(L)+(OMEGA(L-1)-OMEGA(L))*ABSF(DEL(L))/(ABSF(DEL(L)  003000
     1)+ABSF(DEL(L-1)))                                                 003010
      GO TO 32                                                          003020
   56 IF(OMEGA(L)-OMEGAC)21,21,22                                       003030
   57 IF(BB)58,58,50                                                    003040
   58 BB=1.0                                                            003050
   59 IF(OMEGA(L)-OMEGA(L-1)-CR)29,29,60                                003060
   60 M=M-1                                                             003070
      GO TO 3000                                                        003080
   61 IF(GG)70,70,72                                                    003090
   70 IF(DEL(L))77,71,77                                                003100
   71 GG=1.0                                                            003110
      OMEGA(L+1)=OMEGA(L)-CR                                            003120
      GO TO 31                                                          003130
   72 IF(GG-1.0)73,73,74                                                003140
   73 GG=2.0                                                            003150
      HDEL=DEL(L)                                                       003160
      OMEGA(L+1)=OMEGA(L)+2.*CR                                         003170
      GO TO 31                                                          003180
   74 TDEL=DEL(L)*HDEL                                                  003190
      IF(TDEL)75,76,76                                                  003200
   75 OMEGAM(M)=OMEGA(L-2)                                              003210
      GO TO 33                                                          003220
   76 KK=1                                                              003230
      M=M-1                                                             003240
      GO TO 3000                                                        003250
```

```
   77 IF(CC)56,56,54                                                    003260
C                                                                       003270
C      OUTPUT                                                           003280
C                                                                       003290
 3000 IF(NM)400,400,403                                                 003300
  400 PRINT 401                                                         003310
  401 FORMAT (1H1//)                                                    003320
      PRINT 402                                                         003330
 4020FORMAT (15H PROGRAM VIPIPE,35X,17HG. E. FINK  NHA-1,32X,13HFEBRUAR 003340
     1Y 1964///,10X,10SH                MODAL FREQUENCIES OF A PLANAR PIPING 003350
     2 SYSTEM ARE DETERMINED BY AN ITERATIVE PROCEDURE USING THE /2SH ME 003360
     3THOD OF TRANSFER MATRICES.///119H **********                       003370
     4**********                                                        003380
     5**********                  //)                                   003390
  403 IF(NM)4058,4058,404                                               003400
  404 PRINT 405                                                         003410
  405 FORMAT (1H1//)                                                    003420
 4058 NM=1                                                              003430
 4059 IF(JK)4060,4060,4065                                              003440
 4060 PRINT 4070 JJ                                                     003450
 4070OFORMAT (50X,8HPROBLEM I2,//35X,47HIN PLANE MODE FREQUENCIES (RADIA 003460
     1NS PER SECOND)   //45X,4HMODE,15X,10H FREQUENCY /)                003470
      GO TO 4079                                                        003480
 4065 PRINT 4075 JJ                                                     003490
 4075OFORMAT(50X,8HPROBLEM I2,//35X,51HOUT OF PLANE MODE FREQUENCIES (RA 003500
     1DIANS PER SECONC)       //45X,4HMODE,15X,10H FREQUENCY       /)    003510
 4079 MM=M                                                              003520
      PRINT 408 (M,CMEGAM(M),M=1,MM)                                    003530
  408 FORMAT (46X,I2,17X,F14.8 /)                                       003540
      IF(KK)4080,4090,4080                                              003550
 4080 PRINT 4085                                                        003560
 4085OFORMAT(10X,69HPROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITAT 003570
     1ION OF COMPUTER.    /)                                           003580
 4090 IF(AMYK)412,409,412                                               003590
  409 PRINT 411                                                         003600
 4110FORMAT (16X,53H              A LUMPED MASS APPROACH WAS EMPLOYED, A 003610
     1ND )                                                              003620
      GO TO 414                                                         003630
  412 PRINT 413                                                         003640
  413 FORMAT (24X,45HA DISTRIBUTED MASS APPROACH WAS EMPLOYED AND      ) 003650
  414 IF(SD)415,415,420                                                 003660
  415 IF(RI)416,416,418                                                 003670
  416 PRINT 417                                                         003680
 4170FORMAT (2X,67HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA W 003690
     1ERE CONSIDERED. /)                                                003700
      GO TO 426                                                         003710
  418 PRINT 419                                                         003720
 4190FORMAT (3X,63HTHE EFFECTS OF SHEAR DEFLECTION WERE CONSIDERED WHIL 003730
     1E THOSE OF      /3X,25HROTATIONAL INERTIA WERE NEGLECTED.       /) 003740
      GO TO 426                                                         003750
  420 IF(RI)421,421,423                                                 003760
  421 PRINT 422                                                         003770
 4220FORMAT (3X,72HTHE EFFECTS OF ROTATIONAL INERTIA WERE CONSIDERED WH 003780
     1ILE THOSE OF SHEAR          /3X,27HDEFLECTION WERE NEGLECTED.   /) 003790
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      GO TO 423                                                          003800
  423 PRINT 424                                                          003810
 4240FORMAT(5X,66HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WE  003820
     1RE NEGLECTED. /)                                                  003830
  425 PRINT 440                                                         003840
 4400FORMAT(1H1 ********************************************************003850
     1**********************************************************      )003860
      IF(GDABC-1.)4511,452C,4401                                        003870
 4401 PRINT 4402                                                        003880
 44020FORMAT(25H SECTION LENGTH AND SUBSECTION DATA/23X,14HSECTION NUMBE003890
     1R,8X,25HLENGTH OF SECTION(INCHES),5X,21HNUMBER OF SECTIONS      ) 003900
      PRINT 442 (I,TL(I),Z(I),I=1,NS )                                  003910
  443 FORMAT (29X,I3,23X,F7.3,21X,F4.0)                                 003920
      LM=L                                                              003930
      PRINT 441                                                         003940
 4410FORMAT (11H GRAPH DATA /20X,16HITERATION NUMBER,4X,9HFREQUENCY (R  003950
     1ADIANS PER SECOND),3X,25H   VALUE OF FREQUENCY DETERMINANT      ) 003960
      PRINT 444 (L,CMEGA(L),DEL(L),L=1,LM)                              003970
  444 FORMAT (29X,I3,23X,F11.6,18X,E15.8)                               003980
 4520 PRINT 442                                                         003990
  442 FORMAT(1X,10HINPUT DATA /)                                        004000
      PRINT 4521                                                        004010
 45210FORMAT(1X,9HCOMPONENT,4X,8HDIAMETER,5X,14HWALL THICKNESS ,5X,6HLEN 004020
     1GTH,5X,9HRADIUS OF,9X,8HINCLUDED,5X,7HDENSITY,5X,7HELASTIC,5X,5HSH  004030
     2EAR /57X,9HCURVATURE,5X,12HANGLE OF ARC,16X,7HMODULUS,5X,7HMODULUS  004040
     3 /)                                                               004050
      DO 4525 I=1,NS                                                    004060
      IF(P(I))4524,4524,4523                                            004070
 4524 PRINT 4522 I,D(I),GT(I),TL(I),RHO(I),THETR(I),BAMU(I),E(I),G(I)    004080
 45220FORMAT (3X,I3,9X,F7.3,10X,F6.3,7X,F8.3,4X,F8.3,9X,F6.2,8X,F7.2,4X, 004090
     1E8.2,4X,E8.2)                                                     004100
      GO TO 4525                                                        004110
 4523 SH=1.                                                             004120
 4525 CONTINUE                                                          004130
      IF(SH)4530,4530,4526                                              004140
 4526 PRINT 4532                                                        004150
 4532 FORMAT (//1X,7HHANGERS /)                                         004160
      PRINT 4527                                                        004170
 45270FORMAT (2X,9HCOMPONENT,14X,3HCLX,14X,3HCLY,14X,3HCLZ,14X,3HCTX,14X 004180
     1,3HCTY,14X,3HCTZ  /)                                              004190
 4528 DO 4530 I=1,NS                                                    004200
      IF(P(I))4530,4530,4529                                            004210
 4529 PRINT 4531 I,CLX(I),CLY(I),CLZ(I),CTX(I),CTY(I),CTZ(I)            004220
 4531 FORMAT(5X,I3,15X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10. 004230
     12 )                                                               004235
 4530 CONTINUE                                                          004240
      PRINT 448                                                         004250
  448 FORMAT (//20H BOUNDARY CONDITIONS )                               004260
      PRINT 449 (SVI(I,1) I=1,6)                                        004270
  449 FORMAT (5H SVI=6F8.0)                                             004280
      PRINT 451 (SVE(I,1) I=1,6)                                        004290
  451 FORMAT (5H SVE=6F8.0)                                             004300
      IF(NBR)4511,4511,45C2                                             004310
 4502 IF(JK)4505,4505,4515                                              004320
 4505 PRINT 4510((SVIP(I,J) I=1,6) J=2,NNN)                             004330
```

```
 4510 FORMAT (6F3.0)                                                    004340
      GO TO 4511                                                        004350
 4515 PRINT 4517((SVOP(I,J) I=1,6) J=2,NN)                              004360
 4517 FORMAT (6F3.6)                                                    004370
 4511 CONTINUE                                                          004380
      DO 446 M=1,MM                                                     004390
  446 OMEGAM(M)=0.0                                                     004400
      DO 447 L=1,LM                                                     004410
      OMEGA(L)=0.0                                                      004420
  447 DEL(L)=0.0                                                        004430
      DELC=0.0                                                          004440
      OMEGAC=0.0                                                        004450
      IF(MN)453,453,452                                                 004460
  452 MN=0                                                              004470
      L=1                                                               004480
      M=1                                                               004490
      AA=0.0                                                            004500
      BB=0.0                                                            004510
      CC=0.0                                                            004520
      GG=0.0                                                            004530
      KK=0                                                              004540
      ADOM=ADOM                                                         004550
      OR=BR                                                             004560
      OMEGA(1)=OMEGAI                                                   004570
      OMEGAC=OMEGAI                                                     004580
      DOM=ADOM                                                          004590
      GO TO 998                                                         004600
  453 JJ=JJ+1                                                           004610
      DO 454 I=1,NS                                                     004620
      P(I)=0.                                                           004630
  454 Z(I)=0.0                                                          004640
      GO TO 450                                                         004650
      END                                                               004660
C                                                                       004670
C                                                                       004680
      SUBROUTINE SUBSEC (RHO,THETA,D,TL,HM,Z)                           004690
      RHOD=ABSF(RHO)                                                    004700
      IF(HM-5.0)8,8,7                                                   004710
    7 HM=5.0                                                            004720
    8 RA=TL/D                                                           004730
      IF(RHOD-0.)3,3,4                                                  004740
    3 IF(RA-1.)5,5,6                                                    004750
    5 Z=0.                                                              004760
      RETURN                                                            004770
    6 IF(RA-3.) 15,15,16                                                004780
   15 Z=2.0                                                             004790
      RETURN                                                            004800
   16 IF(RA-6.)17,17,18                                                 004810
   17 Z=3.0                                                             004820
      RETURN                                                            004830
   18 IF(HM-3.)1,1,2                                                    004840
    1 Z=6.0                                                             004850
      RETURN                                                            004860
    2 Z=2.0*HM                                                          004870
```

```
      RETURN                                                            004880
    4 RD=RHOD/D                                                         004890
      IF(RL-1.)19,19,23                                                 004900
   19 Z=0                                                               004910
      RETURN                                                            004920
   23 IF(RL-3.)24,24,25                                                 004930
   24 IF(THETA-.44)26,26,27                                             004940
   26 Z=0.                                                              004950
      RETURN                                                            004960
   27 IF(THETA-1.57)28,28,29                                            004970
   28 Z=2.0                                                             004980
      RETURN                                                            004990
   29 Z=3.0                                                             005000
      RETURN                                                            005010
   25 IF(RD-6.)30,30,31                                                 005020
   30 IF(THETA-.44)32,32,33                                             005030
   32 Z=2.0                                                             005040
      RETURN                                                            005050
   33 IF(THETA-1.57)34,34,35                                            005060
   34 Z=3.0                                                             005070
      RETURN                                                            005080
   35 Z=4.0                                                             005090
      RETURN                                                            005100
   31 IF(THETA-.44)36,36,3                                              005110
   36 Z=2.0                                                             005120
      RETURN                                                            005130
      END                                                               005140
C                                                                       005150
C                                                                       005160
      SUBROUTINE DISTM (AMU,AIY,OMEGA,TL,R,D,DI,G,SD,RI,E,FFAC,U)       005170
      DIMENSION U(6,6)                                                  005180
      ARA=(3.141592654/4.0)*(D**2-DI**2)                                005190
      AREA=ARA*FFAC                                                     005200
      BE=TL*OMEGA*SQRTF(AMU/(E*ARA))                                    005210
      P4=R*AMU*TL**4*OMEGA**2                                           005220
      IF(RI)1,1,2                                                       005230
    1 T=R*AMU*(AIY*OMEGA*TL)**2                                         005240
      GO TO 3                                                           005250
    2 T=0.0                                                             005260
    3 IF(SD)4,4,5                                                       005270
    4 S=(AMU*(OMEGA*TL)**2)/(G*AREA)                                    005280
      GO TO 6                                                           005290
    5 S=0.0                                                             005300
    6 SPT=S+T                                                           005310
      VV=SQRTF(P4+(S-T)**2/4.0)                                         005320
      AL1=SQRTF(VV-.5*SPT)                                              005330
      AL2=SQRTF(VV+.5*SPT)                                              005340
      CL=1.0/(AL1**2+AL2**2)                                            005350
      CHL1=COSH(AL1)                                                    005360
      SHL1=SINH(AL1)                                                    005370
      CL2=COSF(AL2)                                                     005380
      SL2=SINF(AL2)                                                     005390
      SBE=SINF(BE)                                                      005400
      CBE=COSF(BE)                                                      005410
```

```
      CC0=CL*(AL2**2*CHL1+AL1**2*CL2)                                   005420
      CC1=CL*((AL**2*SHL1)/AL1+(AL1**2*CL2)/AL2)                        005430
      CC2=CL*(CHL1-CL2)                                                 005440
      CC3=CL*(SHL1/AL1-SL2/AL2)                                         005450
      DO 7 J=1,6                                                        005460
      DO 7 K=1,6                                                        005470
    7 U(J,K)=0.0                                                        005480
      U(1,1)=CPE                                                        005490
      U(1,6)=TL*SPE/(RL*ARA*E)                                          005500
      U(2,2)=CC0-S*CC2                                                  005510
      U(2,3)=TL*(CC1-SPT*CC3)                                           005520
      U(2,4)=R*CC2*TL**2                                                005530
      U(2,5)=(R*TL**3/P4)*((P4+S**2)*CC3-S*CC1)                         005540
      U(3,2)=P4*CC3/TL                                                  005550
      U(3,3)=CC0-T*CC2                                                  005560
      U(3,4)=TL*R*(CC1-T*CC3)                                           005570
      U(3,5)=U(2,4)                                                     005580
      RR=1.0/R                                                          005590
      U(4,2)=P4*RR*CC3/TL**2                                            005600
      U(4,3)=RR*((P4+T**2)*CC3-T*CC1)/TL                                005610
      U(4,4)=U(3,3)                                                     005620
      U(4,5)=U(2,3)                                                     005630
      U(5,2)=P4*RR*(CC1-S*CC3)/TL**3                                    005640
      U(5,3)=U(4,2)                                                     005650
      U(5,4)=U(3,2)                                                     005660
      U(5,5)=U(2,2)                                                     005670
      U(6,1)=-AMU*TL*OMEGA**2*SPE/RE                                    005680
      U(6,6)=CPE                                                        005690
      RETURN                                                            005700
      END                                                               005710
C                                                                       005720
C                                                                       005730
      SUBROUTINE DISTVC (AMU,AIX,AIY,OMEGA,TL,AJT,R,D,DI,G,SD,RI,FFAC,L) 005740
      DIMENSION U(6,6)                                                  005750
      W=1./(G*AJT)                                                      005760
      AREA=(3.14159265354/4.0)*(D**2-DI**2)*FFAC                        005770
      BF=+SQRTF(AMU * ( T L   *AIX*OMEGA)**2*W)                         005780
      PW=R*AMU    * TL **4*OMEGA**2                                     005790
      IF(RI)2,2,3                                                       005800
    2 T=R*AMU*(AIY*OMEGA*TL)**2                                         005810
      GO TO 4                                                           005820
    3 T=0.0                                                             005830
    4 IF(SD)5,5,6                                                       005840
    5 S=(AMU*(OMEGA*TL)**2)/(G*ARLA)                                    005850
      GO TO 7                                                           005860
    6 S=0.0                                                             005870
    7 VV=+SQRTF(P4+(S-T)**2/4.0)                                        005880
      SPT=S+T                                                           005890
      AL1=+SQRTF(VV-.5*SPT)                                             005900
      AL2=+SQRTF(VV+.5*SPT)                                             005910
      CL=1.0/(AL1**2+AL2**2)                                            005920
      CHL1=COSH(AL1)                                                    005930
      SHL1=SINH(AL1)                                                    005940
      CL2=COSF(AL2)                                                     005950
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      SL2=SINF(AL2)                                                     005960
      SPE=SINF(PE)                                                      005970
      CPE=COSF(PE)                                                      005980
      CC0=CL*(AL2**2*(SHL1 + AL1**.*GL2)                                005990
      CC1=CL*((AL2**2*SL1)/AL1 + (AL1**2*SL2)/AL2)                      006000
      CC2=CL*(CHL1-CL2)                                                 006010
      CC3=CL*(SHL1/AL1-SL2/AL2)                                         006020
      DO 1 J=1,4                                                        006030
      DO 1 K=1,6                                                        006040
    1 U(J,K)=0.0                                                        006050
      U(1,1)=CPE                                                        006060
      U(1,2)=TL*W*SPE/PE                                                006070
      U(2,1)=-AMU  * TL  *AIX**2*SPE/BL                                 006080
      U(2,2)=CPE                                                        006090
      U(3,3)=CCC-S*CC2                                                  006100
      U(3,4)= TL *(CC1-SPT*CC3)                                         006110
      U(3,5)=R*CC2*TL**2                                                006120
      U(3,6)=(R*TL**3/PH)*(-S*CC1+(P4+S**2)*CC3)                        006130
      U(4,3)=P4*CC3/TL                                                  006140
      U(4,4)=CCC-T*CC2                                                  006150
      U(4,5)= TL*R*(CC1-T*CC3)                                          006160
      U(4,6)=U(3,5)                                                     006170
      RR=1.0/R                                                          006180
      U(5,3)=P4*RR*CC2/TL**2                                            006190
      U(5,4)=RR*(-T*CC1+(P4+T**2)*CC3)/TL                               006200
      U(5,5)=U(4,4)                                                     006210
      U(5,6)=U(3,4)                                                     006220
      U(6,3)=P4*RR*(CC1-S*CC3)/TL**3                                    006230
      U(6,4)=U(5,3)                                                     006240
      U(6,5)=U(4,5)                                                     006250
      U(6,6)=U(3,5)                                                     006260
      RETURN                                                            006270
      END                                                               006280
C                                                                       006290
C                                                                       006300
      SUBROUTINE SFIELD (FI,TL,P,Z,G,SI,D,E,FFAC,A)                     006310
      DIMENSION A(6,6)                                                  006320
      ARA=(3.1415926654/4.0)*(E**2-DI**2)                               006330
      AREA=ARA*FFAC                                                     006340
      DL =TL/Z                                                          006350
      DO 1 J=2,5                                                        006360
    1 A(1,J)=0.0                                                        006370
      A(1,1)=1.0                                                        006380
      A(1,6)=DL/(G*ARA)                                                 006390
      A(2,1)=0.0                                                        006400
      A(2,2)=1.0                                                        006410
      A(2,5)=DL                                                         006420
      A(2,4)=DL**2*R/2.0                                                006430
      IF(SI)2,2,3                                                       006440
    2 A(2,5)=DL**3*R/6.0-DL/(G*ARLA)                                    006450
      GO TO 4                                                           006460
    3 A(2,5)=DL**3*R/6.0                                                006470
    4 A(2,6)=0.0                                                        006480
      A(3,1)=0.0                                                        006490
```

```
      A(3,2)=0.0                                                        006500
      A(3,3)=1.0                                                        006510
      A(3,4)=DL*R                                                       006520
      A(3,5)=A(2,4)                                                     006530
      A(3,6)=0.0                                                        006540
      DO 5 J=1,6                                                        006550
      A(4,J)=0.0                                                        006560
      A(5,J)=0.0                                                        006570
    5 A(6,J)=0.0                                                        006580
      A(4,4)=1.0                                                        006590
      A(4,5)=DL                                                         006600
      A(5,5)=1.0                                                        006610
      A(6,6)=1.0                                                        006620
      RETURN                                                            006630
      END                                                               006640
C                                                                       006650
C                                                                       006660
      SUBROUTINE SFIELD (DI,TL,AJT,R,Z,G,SCRI,D,FFAC,A)                 006670
      DIMENSION A(6,6)                                                  006680
      AREA=(3.14159265E4/4.0)*((D**2-DI**2)*FFAC                        006690
      DL=TL/Z                                                           006700
      DO 2 J=1,6                                                        006710
      DO 2 K=1,6                                                        006720
    2 A(J,K)=0.0                                                        006730
      A(1,1)=1.0                                                        006740
      A(2,2)=1.0                                                        006750
      A(3,3)=1.0                                                        006760
      A(4,4)=1.0                                                        006770
      A(5,5)=1.0                                                        006780
      A(6,6)=1.0                                                        006790
      IF(SCRI-0.)7,7,8                                                  006800
    7 A(3,6)=DL**3*R/6.0-DL/(G*AREA)                                    006810
      GO TO 1                                                           006820
    8 A(3,6)=DL**3*R/6.0                                                006830
    1 A(1,2)=DL/(G*AJT)                                                 006840
      A(3,4)=DL                                                         006850
      A(3,5)=DL**2*R/2.0                                                006860
      A(4,5)=DL*R                                                       006870
      A(4,6)=A(3,5)                                                     006880
      A(5,6)=DL                                                         006890
      RETURN                                                            006900
      END                                                               006910
C                                                                       006920
C                                                                       006930
      SUBROUTINE CFIELD (R,Z,G,SD,RHO,THETA,D,DI,E,A)                   006940
      DIMENSION A(6,6)                                                  006950
      PHI=THETA/Z                                                       006960
      RD=ABSF(RHO)                                                      006970
      IF(RHO)1,1,2                                                      006980
    1 V=-1.0                                                            006990
      GO TO 3                                                           007000
    2 V=1.0                                                             007010
    3 CP=COSF(PHI)                                                      007020
      SP=SINF(PHI)                                                      007030
```

```
      DO 4 J=1,6                                                        007640
      DO 4 K=1,6                                                        007650
    4 A(J,K)=0.0                                                        007660
      A(1,1)=CP                                                         007670
      A(1,2)=-V*SP                                                      007680
      A(1,3)=-V*RD*(1.0-CP)                                             007690
      A(1,4)=-V*RD**2*R*(PHI-SP)                                        007700
      AR=3.14159265*(D**2-DI**2)/4.0                                    007710
      W=1.0/(E*AR)                                                      007720
      F3=(SP-PHI*CP)*.5                                                 007730
      F4=(PHI+CP+SP)*.5                                                 007740
      F5=(2.0-2.0*CP-PHI*SP)*.5                                         007750
      F6=(2.0*PHI+PHI*CP-3.0*SP)*.5                                     007760
      A(1,5)=V*(RD*W*PHI*SP*.5-R*F5*RD**3)                              007770
      A(1,6)=RD*W*F4+R*F6*RD**2                                         007780
      A(2,1)=V*SP                                                       007790
      A(2,2)=CP                                                         007800
      A(2,3)=RD*SP                                                      007810
      A(2,4)=R*RD**2*(1.0-CP)                                           007820
      A(2,5)=RD*F5*(W+R*RD**2)                                          007830
      A(2,6)=A(1,5)                                                     007840
      A(3,3)=1.0                                                        007850
      A(3,4)=RD*R*PHI                                                   007860
      A(3,5)=A(2,4)                                                     007870
      A(3,6)=A(1,4)                                                     007880
      A(4,4)=1.0                                                        007890
      A(4,5)=A(2,3)                                                     007900
      A(4,6)=A(1,3)                                                     007910
      A(5,5)=CP                                                         007920
      A(5,6)=A(1,2)                                                     007930
      A(6,5)=A(2,1)                                                     007940
      A(6,6)=CP                                                         007950
      RETURN                                                            007960
      END                                                               007970
C                                                                       007980
C                                                                       007990
      SUBROUTINE CFIELD (AJT,R,Z,C,SCRI,RHO,THETA,D,DI,FFAC,A)          008000
      DIMENSION  A(6,6)                                                 008010
      PHI=THETA/Z                                                       008020
      RHOD=RHO                                                          008030
      IF(RHO-0.0)11,11,12                                               008040
   11 V=-1.0                                                            008050
      GO TO 1                                                           008060
   12 V=1.0                                                             008070
    1 CP=COSF(PHI)                                                      008080
      SP=SINF(PHI)                                                      008090
      W=1.0/( C *AJT)                                                   008100
      F1=(PHI*CP+SP)/2.0                                                008110
      F3=(SP-CP*PHI)/2.0                                                008120
      F5=(2.0-2.0*CP-PHI*SP)/2.0                                        008130
      F6=(2.0*PHI+PHI*CP-3.0*SP)/2.0                                    008140
      RHO=ABSF(RHO)                                                     008150
      DO 2 J=1,6                                                        008160
      DO 2 K=1,6                                                        008170
```

```
      2 A(J,K)=C.C                                                      007580
        IF(SORT)13,15,14                                                007590
     13 AREA=(3.14159265/4.0)*((**2-DI**2)*FFAC                         007600
        SD=RHO*THETA/(S*Z*AREA)                                         007610
        GO TO 15                                                        007620
     14 SD=0.0                                                          007630
     15 A(1,1)=CP                                                       007640
        A(1,2)=(W*RHO*F1)-(RHO*F3*R)                                    007650
        A(1,4)=V*SP                                                     007660
        A(1,5)=V*(W+R)*(RHO*PHI*SP)/2.0                                 007670
        A(1,6)=V*(W*R)*(RHO**2)*F2                                      007680
        A(2,2)=CP                                                       007690
        A(2,5)=V*SP                                                     007700
        A(2,6)=V*RHO*(1.0-CP)                                           007710
        A(3,1)=-A(2,6)                                                  007720
        A(3,2)=-1.0*A(1,6)                                              007730
        A(3,3)=1.C                                                      007740
        A(3,4)=RHO*SP                                                   007750
        A(3,5)=((R*(RHO**2)*PHI*SP)/2.C)-W*RHO**2*F5                    007760
        A(3,6)=(R*RHO**3*F3)-(W*F6*RHO**2)-SD                           007770
        A(4,1)=-V*SP                                                    007780
        A(4,2)=-V*(W+R)*RHO*PHI*SP/2.0                                  007790
        A(4,4)=CP                                                       007800
        A(4,5)=(R*RHO*F1)-(W*RHO*F3)                                    007810
        A(4,6)=A(3,5)                                                   007820
        A(5,2)=-V*SP                                                    007830
        A(5,5)=CP                                                       007840
        A(5,6)=A(3,4)                                                   007850
        A(6,6)=1.C                                                      007860
        RHO=RHOC                                                        007870
        RETURN                                                          007880
        END                                                             007890
C                                                                       007900
C                                                                       007910
        SUBROUTINE PCINT (AMU,AIY,OMEGA,TL,Z,RI,B)                      007920
        DIMENSION B(6,6)                                                007930
        AM=AMU*TL/(Z-1.)                                                007940
        DO 1 J=1,6                                                      007950
        DO 1 K=1,6                                                      007960
      1 B(J,K)=0.0                                                      007970
        B(1,1)=1.0                                                      007980
        B(2,2)=1.0                                                      007990
        B(3,3)=1.0                                                      008000
        IF(RI)2,2,3                                                     008010
      2 B(4,3)=-AM*(AIY*OMEGA)**2                                       008020
      3 B(4,4)=1.0                                                      008030
        B(5,2)=AM*OMEGA**2                                              008040
        B(5,5)=1.0                                                      008050
        B(6,1)=-B(5,2)                                                  008060
        B(6,6)=1.0                                                      008070
        RETURN                                                          008080
        END                                                             008090
C                                                                       008100
C                                                                       008110
```

```
      SUBROUTINE PCINC (AMU,AIX,AIY,OMEGA,TL,Z,SDRI,P)                  008120
      DIMENSION P(6,6)                                                  008130
      AM=AMU*TL/(Z-1.0)                                                 008140
      DO 1 J=1,6                                                        008150
      DO 1 K=1,6                                                        008160
    1 P(J,K)=0.0                                                        008170
      P(1,1)=1.0                                                        008180
      P(2,1)=-AM*(AIX*OMEGA)**2                                         008190
      P(2,2)=1.0                                                        008200
      P(3,3)=1.0                                                        008210
      P(4,4)=1.0                                                        008220
      IF(SDRI-0.)9,9,10                                                 008230
    9 P(5,4)=-AM*(AIY*OMEGA)**2                                         008240
      GO TO 11                                                          008250
   10 P(5,4)=0                                                          008260
   11 P(5,5)=1.0                                                        008270
      P(6,3)=AM*OMEGA**2                                                008280
      P(6,6)=1.0                                                        008290
      RETURN                                                            008300
      END                                                               008310
C                                                                       008320
C                                                                       008330
      SUBROUTINE MATMUL (A,B,Z,U)                                       008340
      DIMENSION A(6,6),B(6,6),C(6,6),D(6,6),U(6,6)                      008350
      DO 1 J=1,6                                                        008360
      DO 1 K=1,6                                                        008370
      C(J,K)=0.0                                                        008380
      DO 1 L=1,6                                                        008390
    1 C(J,K)=C(J,K)+B(J,L)*A(L,K)                                       008400
      N=Z-1.8                                                           008410
      DO 2 J=1,6                                                        008420
      DO 2 K=1,6                                                        008430
    2 U(J,K)=C(J,K)                                                     008440
      IF(Z-2.0)6,6,3                                                    008450
    3 DO 5 M=1,N                                                        008460
      DO 4 J=1,6                                                        008470
      DO 4 K=1,6                                                        008480
      D(J,K)=0.0                                                        008490
      DO 4 L=1,6                                                        008500
    4 D(J,K)=D(J,K)+C(J,L)*U(L,K)                                       008510
      DO 5 J=1,6                                                        008520
      DO 5 K=1,6                                                        008530
    5 U(J,K)=D(J,K)                                                     008540
    6 DO 7 J=1,6                                                        008550
      DO 7 K=1,6                                                        008560
      D(J,K)=0.0                                                        008570
      DO 7 L=1,6                                                        008580
    7 D(J,K)=D(J,K)+A(J,L)*U(L,K)                                       008590
      DO 8 J=1,6                                                        008600
      DO 8 K=1,6                                                        008610
    8 U(J,K)=D(J,K)                                                     008620
      RETURN                                                            008630
      END                                                               008640
C                                                                       008650
```

```
C                                                                       008660
      SUBROUTINE RIGID (AMU,TL,OMEGA,AIY,SRI,U)                         008670
      DIMENSION U(6,6)                                                  008680
      AM=AMU*TL                                                         008690
      DO 1 J=1,6                                                        008700
      DO 1 K=1,6                                                        008710
    1 U(J,K)=0.0                                                        008720
      U(1,1)=1.0                                                        008730
      U(6,1)=-AM*OMEGA**2                                               008740
      U(6,6)=1.0                                                        008750
      U(2,2)=1.0                                                        008760
      U(2,5)=TL                                                         008770
      U(3,3)=1.0                                                        008780
      U(4,2)=AM*TL*OMEGA**2/2.0                                         008790
      IF(SRI)2,2,3                                                      008800
    2 U(4,5)=AM*OMEGA**2*(TL**2/6.0-AIY**2)                             008810
      GO TO 4                                                           008820
    3 U(4,4)=AM*(OMEGA*TL)**2/6.0                                       008830
    4 U(4,4)=1.0                                                        008840
      U(4,5)=TL                                                         008850
      U(5,2)=-U(6,1)                                                    008860
      U(5,3)=U(4,2)                                                     008870
      U(5,5)=1.0                                                        008880
      RETURN                                                            008890
      END                                                               008900
C                                                                       008910
C                                                                       008920
      SUBROUTINE RIGIC (AMU,TL,AIX,OMEGA,AIY,AJT,SRI,U)                 008930
      DIMENSION U(6,6)                                                  008940
      DO 2 J=1,6                                                        008950
      DO 2 K=1,6                                                        008960
    2 U(J,K)=0.0                                                        008970
      AM=AMU*TL                                                         008980
      U(1,1)=1.0                                                        008990
      U(2,1)=-AMU*TL*(AIX*OMEGA)**2                                     009000
      U(2,2)=1.0                                                        009010
      U(3,3)=1.0                                                        009020
      U(3,4)=TL                                                         009030
      U(4,4)=1.0                                                        009040
      U(5,3)=AM*TL*OMEGA**2/2.0                                         009050
      IF(SRI-0.) 30,30,31                                               009060
   30 U(5,4)=AM*OMEGA**2*(TL**2/6.0-AIY**2)                             009070
      GO TO 1                                                           009080
   31 U(5,4)=AM*OMEGA**2*(TL**2/6.0)                                    009090
    1 U(5,5)=1.0                                                        009100
      U(5,6)=TL                                                         009110
      U(6,3)=AM*OMEGA**2                                                009120
      U(6,4)=U(5,3)                                                     009130
      U(6,6)=1.0                                                        009140
      RETURN                                                            009150
      END                                                               009160
C                                                                       009170
C                                                                       009180
      SUBROUTINE STIFCC (THETA,RFC,U)                                   009190
```

```
      DIMENSION U(6,6)                                              009500
      DO 1 J=1,6                                                    009510
      DO 1 K=1,6                                                    009520
    1 U(J,K)=0.0                                                    009530
      IF(RHO)2,2,3                                                  009540
    2 V=-1.0                                                        009550
      GO TO 4                                                       009560
    3 V=1.0                                                         009570
    4 CT=COSF(THETA)                                                009580
      ST=SINF(THETA)                                                009590
      U(1,1)=CT                                                     009600
      U(1,2)=-V*ST                                                  009610
      U(2,1)=V*ST                                                   009620
      U(2,2)=CT                                                     009630
      U(3,3)=1.0                                                    009640
      U(4,4)=1.0                                                    009650
      U(5,5)=CT                                                     009660
      U(5,6)=-V*ST                                                  009670
      U(6,5)=V*ST                                                   009680
      U(6,6)=CT                                                     009690
      RETURN                                                        009700
      END                                                           009710
C                                                                   009720
C                                                                   009730
      SUBROUTINE STIFCO (THETA,RHO,U)                               009740
      DIMENSION U(6,6)                                              009750
      IF(RHO)1,1,2                                                  009760
    1 V=-1.0                                                        009770
      GO TO 3                                                       009780
    2 V=1.0                                                         009790
    3 CT=COSF(THETA)                                                009800
      ST=SINF(THETA)                                                009810
      DO 4 J=1,6                                                    009820
      DO 4 K=1,6                                                    009830
    4 U(J,K)=0.0                                                    009840
      U(1,1)=CT                                                     009850
      U(1,4)=ST*V                                                   009860
      U(2,2)=CT                                                     009870
      U(2,5)=ST*V                                                   009880
      U(3,3)=1.0                                                    009890
      U(4,1)=-ST*V                                                  009900
      U(4,4)=CT                                                     009910
      U(5,2)=-ST*V                                                  009920
      U(5,5)=CT                                                     009930
      U(6,6)=1.0                                                    009940
      RETURN                                                        009950
      END                                                           009960
C                                                                   009970
C                                                                   009980
C                                                                   009990
      SUBROUTINE HANGER (CLX,CLY,CTZ,L)                             009700
      DIMENSION U(6,6)                                              009710
      DO 1 J=1,6                                                    009720
      DO 1 K=1,6                                                    009730
```

```
    1 U(J,K)=0.0                                                          009740
      U(1,1)=1.0                                                          009750
      U(2,2)=1.0                                                          009760
      U(3,3)=1.0                                                          009770
      U(4,3)=CTZ                                                          009780
      U(4,4)=1.0                                                          009790
      U(5,2)=-CLY                                                         009800
      U(5,5)=1.0                                                          009810
      U(6,1)=CLX                                                          009820
      U(6,6)=1.0                                                          009830
      RETURN                                                              009840
      END                                                                 009850
C                                                                         009860
C                                                                         009870
      SUBROUTINE PANGLC (CTX,CLZ,CTY,L)                                   009880
      DIMENSION U(6,6)                                                    009890
      DO 1 J=1,6                                                          009900
      DO 1 K=1,6                                                          009910
    1 U(J,K)=0.0                                                          009920
      U(1,1)=1.0                                                          009930
      U(2,1)=CTX                                                          009940
      U(2,2)=1.0                                                          009950
      U(3,3)=1.0                                                          009960
      U(4,4)=1.0                                                          009970
      U(5,4)=CTY                                                          009980
      U(5,5)=1.0                                                          009990
      U(6,3)=-CLZ                                                         010000
      U(6,6)=1.0                                                          010010
      RETURN                                                              010020
      END                                                                 010030
C                                                                         010040
C                                                                         010050
      SUBROUTINE FRANCH (SVE,VV,PHI,MI,U)                                 010060
     ODIMENSIONSVE(6,22),VV(6,6),U(6,6),R1(25,25),LL(25),MM(25),R2(3,3),  010070
     1R(3,3),G1(3,3),G2(3,3)                                              010080
      M=1                                                                 010090
      DO 2 N=1,6                                                          010100
      IF(SVE(N,MI))3,3,1                                                  010110
    1 DO 2 L=1,6                                                          010120
    2 VV(L,M)=VV(L,N)                                                     010130
      M=M+1                                                               010140
    3 CONTINUE                                                            010150
      DO 4 L=1,3                                                          010160
      R1(1,L)=VV(1,L)                                                     010170
      R1(2,L)=VV(2,L)                                                     010180
      R1(3,L)=VV(3,L)                                                     010190
      R2(1,L)=VV(4,L)                                                     010200
      R2(2,L)=VV(5,L)                                                     010210
    4 R2(3,L)=VV(6,L)                                                     010220
      CALL INVERT (R1,3,D,LL,MM)                                          010230
      DO 5 J=1,3                                                          010240
      DO 5 K=1,3                                                          010250
      R(J,K)=0.0                                                          010260
      DO 5 L=1,3                                                          010270
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    5 R(J,K)=R(J,K)+R2(J,L)*R1(L,K)                                    010280
      SP=SINF(PHI)                                                     010290
      CP=COSF(PHI)                                                     010300
      G1(1,1)=CP                                                       010310
      G1(1,2)=-SP                                                      010320
      G1(1,3)=0.                                                       010330
      G1(2,1)=SP                                                       010340
      G1(2,2)=CP                                                       010350
      G1(2,3)=0.                                                       010360
      G1(3,1)=0.                                                       010370
      G1(3,2)=0.                                                       010380
      G1(3,3)=1.                                                       010390
      G2(1,1)=1.                                                       010400
      G2(1,2)=0.                                                       010410
      G2(1,3)=0.                                                       010420
      G2(2,1)=0.                                                       010430
      G2(2,2)=CP                                                       010440
      G2(2,3)=-SP                                                      010450
      G2(3,1)=0.                                                       010460
      G2(3,2)=SP                                                       010470
      G2(3,3)=CP                                                       010480
      DO 6 J=1,3                                                       010490
      DO 6 K=1,3                                                       010500
      R1(J,K)=0.0                                                      010510
      DO 6 L=1,3                                                       010520
    6 R1(J,K)=R1(J,K)+G2(J,L)*R(L,K)                                   010530
      DO 7 J=1,3                                                       010540
      DO 7 K=1,3                                                       010550
      R(J,K)=0.0                                                       010560
      DO 7 L=1,3                                                       010570
    7 R(J,K)=R(J,K)+R1(J,L)*G1(L,K)                                    010580
      DO 8 J=1,6                                                       010590
      DO 8 K=1,6                                                       010600
    8 U(J,K)=0.0                                                       010610
      U(1,1)=1.0                                                       010620
      U(2,2)=1.0                                                       010630
      U(3,3)=1.0                                                       010640
      U(4,4)=1.0                                                       010650
      U(5,5)=1.0                                                       010660
      U(6,6)=1.0                                                       010670
      DO 9 L=1,3                                                       010680
      U(4,L)=R(1,L)                                                    010690
      U(5,L)=R(2,L)                                                    010700
    9 U(6,L)=R(3,L)                                                    010710
      RETURN                                                           010720
      END                                                              010730
C                                                                      010740
C                                                                      010750
      SUBROUTINE STAVEC(SV,N,BC)                                       010760
      DIMENSION BC(6,22)                                               010770
      DO 10 J=1,6                                                      010780
   10 BC(J,N)=0.                                                       010790
      IF(SV-1.)1,1,2                                                   010800
    1 BC(4,N)=1.0                                                      010810
```

```
      PC(5,N)=1.0                                                       010820
      PC(6,N)=1.0                                                       010830
      RETURN                                                            010840
    2 IF(SV-2.)3,3,4                                                    010850
    3 PC(1,N)=1.0                                                       010860
      PC(2,N)=1.0                                                       010870
      PC(3,N)=1.0                                                       010880
      RETURN                                                            010890
    4 IF(SV-3.)5,5,6                                                    010900
    5 PC(3,N)=1.0                                                       010910
      PC(5,N)=1.0                                                       010920
      PC(6,N)=1.0                                                       010930
      RETURN                                                            010940
    6 IF(SV-4.)7,7,8                                                    010950
    7 PC(1,N)=1.0                                                       010960
      PC(2,N)=1.0                                                       010970
      PC(5,N)=1.0                                                       010980
      RETURN                                                            010990
    8 PC(2,N)=1.0                                                       011000
      PC(3,N)=1.0                                                       011010
      PC(6,N)=1.0                                                       011020
      RETURN                                                            011030
      END                                                               011040
C                                                                       011050
C                                                                       011060
      SUBROUTINE BRANCC (SVP,VV,PFI,MI,U)                               011070
     ODIMENSIONSVB(6,22),U(6,6),VV(6,6),V(6,6),R1(25,25),R2(3,3),R(3,3),  011080
     1G1(3,3),G2(3,3),S(3,3),LL(25),MM(25),SR(6,22),UV(6,6)             011090
      M=1                                                               011100
      SR(1,MI)=SVB(1,MI)                                                011110
      SR(2,MI)=SVB(3,MI)                                                011120
      SR(3,MI)=SVB(4,MI)                                                011130
      SR(4,MI)=SVB(2,MI)                                                011140
      SR(5,MI)=SVB(5,MI)                                                011150
      SR(6,MI)=SVB(6,MI)                                                011160
      UV(1,1)=VV(1,1)                                                   011170
      UV(1,2)=VV(1,3)                                                   011180
      UV(1,3)=VV(1,4)                                                   011190
      UV(1,4)=VV(1,2)                                                   011200
      UV(1,5)=VV(1,5)                                                   011210
      UV(1,6)=VV(1,6)                                                   011220
      UV(2,1)=VV(3,1)                                                   011230
      UV(2,2)=VV(3,3)                                                   011240
      UV(2,3)=VV(3,4)                                                   011250
      UV(2,4)=VV(3,2)                                                   011260
      UV(2,5)=VV(3,5)                                                   011270
      UV(2,6)=VV(3,6)                                                   011280
      UV(3,1)=VV(4,1)                                                   011290
      UV(3,2)=VV(4,3)                                                   011300
      UV(3,3)=VV(4,4)                                                   011310
      UV(3,4)=VV(4,2)                                                   011320
      UV(3,5)=VV(4,5)                                                   011330
      UV(3,6)=VV(4,6)                                                   011340
      UV(4,1)=VV(2,1)                                                   011350
```

```
      UV(4,2)=VV(3,3)                                                   011550
      UV(4,3)=VV(3,4)                                                   011560
      UV(4,4)=VV(3,2)                                                   011570
      UV(4,5)=VV(3,5)                                                   011580
      UV(4,6)=VV(3,6)                                                   011590
      UV(5,1)=VV(5,1)                                                   011600
      UV(5,2)=VV(5,5)                                                   011610
      UV(5,3)=VV(5,4)                                                   011620
      UV(5,4)=VV(5,2)                                                   011630
      UV(5,5)=VV(5,3)                                                   011640
      UV(5,6)=VV(5,6)                                                   011650
      UV(6,1)=VV(6,1)                                                   011660
      UV(6,2)=VV(6,3)                                                   011670
      UV(6,3)=VV(6,4)                                                   011680
      UV(6,4)=VV(6,2)                                                   011690
      UV(6,5)=VV(6,5)                                                   011700
      UV(6,6)=VV(6,6)                                                   011710
      DO 5 K=1,6                                                        011720
      IF(SR(N,M))5,3,1                                                  011730
1     DO 2 L=1,6                                                        011740
2     UV(L,M)=UV(L,K)                                                   011750
      M=M+1                                                             011760
3     CONTINUE                                                          011770
      DO 4 K=1,3                                                        011780
      R1(1,K)=UV(1,K)                                                   011790
      R1(2,K)=UV(2,K)                                                   011800
      R1(3,K)=UV(3,K)                                                   011810
      R2(1,K)=UV(4,K)                                                   011820
      R2(2,K)=UV(5,K)                                                   011830
4     R2(3,K)=UV(6,K)                                                   011840
      CALL INVERT (R1,3,D,LL,MM)                                        011850
      DO 5 J=1,3                                                        011860
      DO 5 K=1,3                                                        011870
      R(J,K)=0.0                                                        011880
      DO 5 L=1,3                                                        011890
5     R(J,K)=R(J,K)+R2(J,L)*R1(L,K)                                     011900
      SP=SINF(PHI)                                                      011910
      CP=COSF(PHI)                                                      011920
      G1(1,1)=CP                                                        011930
      G1(1,2)=0.                                                        011940
      G1(1,3)=-SP                                                       011950
      G1(2,1)=0.                                                        011960
      G1(2,2)=1.                                                        011970
      G1(2,3)=0.                                                        011980
      G1(3,1)=SP                                                        011990
      G1(3,2)=0.                                                        012000
      G1(3,3)=CP                                                        012010
      G2(1,1)=CP                                                        012020
      G2(1,2)=SP                                                        012030
      G2(1,3)=0.                                                        012040
      G2(2,1)=-SP                                                       012050
      G2(2,2)=CP                                                        012060
      G2(2,3)=0.                                                        012070
      G2(3,1)=0.                                                        012080
```

```
      G2(3,2)=0.                                                    011900
      G2(3,3)=1.                                                    011910
      DO 6 J=1,3                                                    011920
      DO 6 K=1,3                                                    011930
      R1(J,K)=0.0                                                   011940
      DO 6 L=1,3                                                    011950
    6 R1(J,K)=R1(J,K)+C2(J,L)*R(L,K)                                011960
      DO 7 J=1,3                                                    011970
      DO 7 K=1,3                                                    011980
      S(J,K)=0.0                                                    011990
      DO 7 L=1,3                                                    012000
    7 S(J,K)=S(J,K)+R1(J,L)*C1(L,K)                                 012010
      DO 8 J=1,6                                                    012020
      DO 8 K=1,6                                                    012030
    8 U(J,K)=0.0                                                    012040
      U(1,1)=1.0                                                    012050
      U(2,1)=S(1,1)                                                 012060
      U(2,2)=1.0                                                    012070
      U(2,3)=S(1,2)                                                 012080
      U(2,4)=S(1,3)                                                 012090
      U(3,3)=1.0                                                    012100
      U(4,4)=1.0                                                    012110
      U(5,1)=S(2,1)                                                 012120
      U(5,2)=S(2,2)                                                 012130
      U(5,4)=S(2,3)                                                 012140
      U(5,5)=1.0                                                    012150
      U(6,1)=S(3,1)                                                 012160
      U(6,2)=S(3,2)                                                 012170
      U(6,4)=S(3,3)                                                 012180
      U(6,6)=1.0                                                    012190
      RETURN                                                        012200
      END                                                           012210
C                                                                   012220
C                                                                   012230
      SUBROUTINE STAVEC (SV,N,BC)                                   012240
      DIMENSION BC(6,22)                                            012250
      DO 10 L=1,6                                                   012260
   10 BC(L,N)=0.0                                                   012270
      IF(SV-1.0)1,1,2                                               012280
    1 BC(2,N)=1.0                                                   012290
      BC(5,N)=1.0                                                   012300
      BC(6,N)=1.0                                                   012310
      RETURN                                                        012320
    2 IF(SV-2.0)3,3,4                                               012330
    3 BC(1,N)=1.0                                                   012340
      BC(3,N)=1.0                                                   012350
      BC(4,N)=1.0                                                   012360
      RETURN                                                        012370
    4 IF(SV-3.0)5,5,6                                               012380
    5 BC(2,N)=1.0                                                   012390
      BC(4,N)=1.0                                                   012400
      BC(6,N)=1.0                                                   012410
      RETURN                                                        012420
    6 IF(SV-4.0)7,7,5                                               012430
```

```
     7 B(I,J)=1.0                                                       012440
       B(K,K)=1.0                                                       012450
       B(J,K)=1.0                                                       012460
       RETURN                                                           012470
       END                                                              012480
C                                                                       012490
C                                                                       012500
       SUBROUTINE INVERT (A,N,B,L,M)                                    012510
C      PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX                  012520
C                                                                       012530
       DIMENSION A(25,25),L(25),M(25)                                   012540
C      SEARCH FOR LARGEST ELEMENT                                       012550
       D=1.0                                                            012560
       DO80 K=1,N                                                       012570
       L(K)=K                                                           012580
       M(K)=K                                                           012590
       BIGA=A(K,K)                                                      012600
       DO20 I=K,N                                                       012610
       DO20 J=K,N                                                       012620
       IF(ABSF(BIGA)-ABSF(A(I,J))) 10,20,20                             012630
    10 BIGA=A(I,J)                                                      012640
       L(K)=I                                                           012650
       M(K)=J                                                           012660
    20 CONTINUE                                                         012670
C      INTERCHANGE ROWS                                                 012680
       J=L(K)                                                           012690
       IF(L(K)-K) 35,35,25                                              012700
    25 DO30 I=1,N                                                       012710
       HOLD=-A(K,I)                                                     012720
       A(K,I)=A(J,I)                                                    012730
    30 A(J,I)=HOLD                                                      012740
C      INTERCHANGE COLUMNS                                              012750
    35 I=M(K)                                                           012760
       IF(M(K)-K) 45,45,37                                              012770
    37 DO40 J=1,N                                                       012780
       HOLD=-A(J,K)                                                     012790
       A(J,K)=A(J,I)                                                    012800
    40 A(J,I)=HOLD                                                      012810
C      DIVIDE COLUMN BY MINUS PIVOT                                     012820
    45 DO55 I=1,N                                                       012830
    46 IF(I-K)50,55,50                                                  012840
    50 A(I,K)=A(I,K)/(-A(K,K))                                          012850
    55 CONTINUE                                                         012860
C      REDUCE MATRIX                                                    012870
       DO65 I=1,N                                                       012880
       DO65 J=1,N                                                       012890
    56 IF(I-K) 57,65,57                                                 012900
    57 IF(J-K) 60,65,60                                                 012910
    60 A(I,J)=A(I,K)*A(K,J)+A(I,J)                                      012920
    65 CONTINUE                                                         012930
C      DIVIDE ROW BY PIVOT                                              012940
       DO75 J=1,N                                                       012950
    68 IF(J-K)70,75,70                                                  012960
    70 A(K,J)=A(K,J)/A(K,K)                                             012970
```

```
   75 CONTINUE                                                          012960
C     CONTINUE PRODUCT OF PIVOTS                                       012970
      D=D*A(K,K)                                                       013000
C     REPLACE PIVOT BY RECIPROCAL                                      013010
      A(K,K)=1.0/A(K,K)                                                013020
   80 CONTINUE                                                         013030
C     FINAL ROW AND COLUMN INTERCHANGE                                 013040
      K=N                                                              013050
  100 K=(K-1)                                                          013060
      IF(K) 150,150,103                                                013070
  103 I=L(K)                                                           013080
      IF(I-K) 120,120,105                                              013090
  105 DO 110 J=1,N                                                     013100
      HOLD=A(J,K)                                                      013110
      A(J,K)=-A(J,I)                                                   013120
  110 A(J,I)=HOLD                                                      013130
  120 J=M(K)                                                           013140
      IF(J-K) 100,100,125                                              013150
  125 DO 130 I=1,N                                                     013160
      HOLD=A(I,J)                                                      013170
      A(K,I)=-A(J,I)                                                   013180
  130 A(J,I)=HOLD                                                      013190
      GO TO 100                                                        013200
  150 RETURN                                                           013210
      END                                                              013220
C                                                                      013230
C                                                                      013240
      SUBROUTINE FINMAT (U,UU,I,V)                                     013250
      DIMENSION U(6,6),UU(6,6),V(6,6)                                  013260
      IF(I-1)61,6 ,61                                                  013270
   62 DO 70 J=1,6                                                      013280
      DO 70 K=1,6                                                      013290
   70 V(J,K)=UU(J,K)                                                   013300
      RETURN                                                           013310
   61 DO 71 J=1,6                                                      013320
      DO 71 K=1,6                                                      013330
      V(J,K)=0.0                                                       013340
      DO 71 L=1,6                                                      013350
   71 V(J,K)=V(J,K)+U(J,L)*UU(L,K)                                     013360
      RETURN                                                           013370
      END                                                              013380
C                                                                      013390
C                                                                      013400
      SUBROUTINE FINBRA (U,VV,NNBR,V)                                  013410
      DIMENSION U(6,6),VV(6,6),V(6,6)                                  013420
      IF(NNBR-1)1,1,3                                                  013430
    1 DO 2 J=1,6                                                       013440
      DO 2 K=1,6                                                       013450
    2 V(J,K)=U(J,K)                                                    013460
      RETURN                                                           013470
    3 DO 4 J=1,6                                                       013480
      DO 4 K=1,6                                                       013490
      V(J,K)=0.0                                                       013500
      DO 4 L=1,6                                                       013510
```

```
    4 V(J,K)=V(J,K)+U(J,L)*VV(L,K)                                      013520
      RETURN                                                            013530
      END                                                               013540
C                                                                       013550
C                                                                       013560
      SUBROUTINE DELMA(UU,SVI,SVE,V)                                    013570
      DIMENSION UU(6,6),VV(6,6),V(6,6),SVI(6,1),SVE(6,1)                013580
      M=0                                                               013590
      DO 90 N=1,6                                                       013600
      IF(SVI(N,1))1,90,1                                                013610
    1 M=M+1                                                             013620
      DO 91 J=1,6                                                       013630
   91 VV(J,M)=UU(J,N)                                                   013640
   90 CONTINUE                                                          013650
      L=0                                                               013660
      DO 92 N=1,6                                                       013670
      IF(SVE(N,1))92,2,92                                               013680
    2 L=L+1                                                             013690
      DO 93 K=1,3                                                       013700
   93 V(L,K)=VV(N,K)                                                    013710
   92 CONTINUE                                                          013720
      RETURN                                                            013730
      END                                                               013740
      END                                                               013750
```

APPENDIX E

ACCURACY OF METHOD

E.1 General

Appendix E contains the vibration analyses of the several typical piping configurations to which reference is made in Chapter 3. The analyses are grouped according to the purpose for which they were performed. The first group was performed to establish the accuracy of the method embodied in the program VIPIPE, the second and third to establish its integrity.

E.2 Method Accuracy Analyses

E.21 Systems Analyzed (straight sections)

System Parameters	Systems		
	1.	2.	3.
end conditions	fixed-fixed	fixed-free	fixed-dropped
length (in.)	200	200	200
diameter (in.)	2.0	2.0	2.0
wall thickness (in.)	.125	.125	.125
density (lb./cu.ft.)	1172	1172	1172
shear modulus (psi)	$30 \cdot 10^6$	$30 \cdot 10^6$	$30 \cdot 10^6$
elastic modulus (psi)	$12 \cdot 10^6$	$12 \cdot 10^6$	$12 \cdot 10^6$

E.22 Comparison of Natural Frequencies[1]

a. System 1

Mode number	frequency (rad./sec.) VIPIPE	Comparison Values	Difference (%)
1	48.53300867	48.533126	.00024
2	133.78346278	133.78345	.00000
3	262.26843097	262.26886	.00016
4	433.54478122	433.54393	.00019
5	647.62290622	647.63991	.0026

b. System 2

Mode number	frequency (rad./sec.) VIPIPE	Comparison Values	Difference (%)
1	7.62356485	7.6270709	.04595
2	47.79657598	47.797761	.00247
3	133.8363688	133.83635	.00001
4	262.26528107	262.26544	.00006
5	433.54435149	433.54415	.00004

c. System 3

Mode number	frequency (rad./sec.) VIPIPE	Comparison Values	Difference (%)
1	33.44525745	33.445600	.00102
2	108.38602303	108.38599	.00003
3	226.1387715	226.13868	.00000
4	386.71069206	386.71057	.00003
5	590.10235871	590.10164	.00012

[1]Sources of comparison frequencies for problems 1 through 3 are given in E.23.

E.23 Source of Comparison Frequencies for Systems 1 through 3

The governing fourth order equation of a homogeneous single component straight section system

$$y^{iv} = \frac{\mu\omega^2}{EJ}y$$

was solved to give

$$y = a\cdot\cos\alpha x + b\cdot\cosh\alpha x + c\cdot\sin\alpha x + d\cdot\sinh\alpha x$$

Upon applying boundary conditions for problem 1 ($y=y'=0$ at $x=0$, and $x=L$) and simplifying it was found that the eigenvalues of the system must satisfy

$$\cos\alpha\cosh\alpha = 1 \qquad \text{(E.23.1)}$$

where

$$\alpha = \sqrt[4]{\frac{\mu\omega^2}{EJ}}\,L$$

Upon applying the boundary conditions of problem 2 ($y=y'=0$ at $x=0$, $y''=y'''=0$ at $x=L$) it was found that the eigenvalues of the system must satisfy

$$\cos\alpha\cosh\alpha = -1 \qquad \text{(E.23-2)}$$

Upon applying the boundary conditions of problem 3 ($y=y'=0$ at $x=0$, $y=y''=0$ at $x=L$) it was found that the eigenvalues of the system must satisfy.

$$\sin\alpha\cosh\alpha - \cos\alpha\sinh\alpha = 0 \qquad \text{(E.23-3)}$$

The roots of equations E.23-1, E.23-2 and E.23-3, and from them the first five natural frequencies of the systems, were obtained through the use of a digital computer.

E.24 System Analyzed (section of constant curvature)

System parameters:

radius of curvature (in.)	50
included angle of arc	270^{o}
diameter (in.)	2
wall thickness (in.)	.125
elastic modulus	$30 \cdot 10^{6}$ psi
shear modulus	$12 \cdot 10^{6}$ psi
density (lbs./cu.-ft.)	556

end conditions: fixed-fixed

Results:

in-plane vibrations[2]

Mode number	frequency (rad./sec.) VIPIPE	Comparison Values	Difference (%)
1	73.32973072	73.6	.23

out-of-plane vibrations

Mode number	frequency (rad./sec.) VIPIPE	Comparison Values[1]	Difference (%)
1	38.87735138	39.8	2.3

E.3 Method Integrity Analyses

The first group of analyses considered here constitutes a mirror image check on system integrity. Four configurations were considered, and several mathematical models were used. The second group consists of two analyses of

---

[1] This value was obtained using a curve in reference 2 which was constructed through the use of a modified Rayleigh method. In the words of the reference; "...laboratory tests have shown the curves to be substantially correct."

[2] Comparison values obtained by employing the equation $\omega = F(\alpha)\frac{2}{R^2}\sqrt{\frac{EJ}{\mu R^4}}$ after obtaining F from curves in ref. 2.

the same single branch system wherein first one member then another is considered to be the branch.

E.31 Mirror Image Analyses

a. System 1

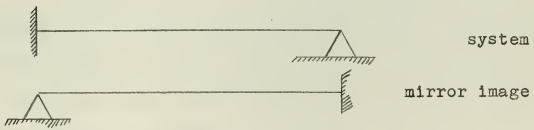
System parameters: See system 2 E.21

Mathematical model: Distributed mass with the effects of shear deflection and rotational inertia considered.

Results:

Mode number	frequency (rad./sec.)		Difference (%)
	System	Mirror Image	
1	7.622895556	7.62289556	.00000
2	47.76681871	47.76681872	.00000
3	133.63614681	133.63614691	.00000
4	261.54431793	261.54431024	.00000
5	431.64844506	431.64917860	.00016

b. System 2



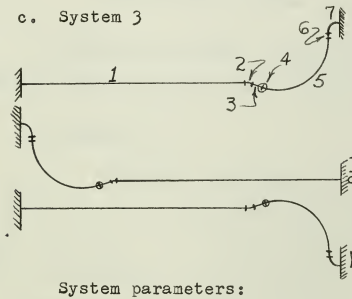
System parameters: See system 3, E.21

Mathematical model: Distributed mass with the effects of shear deflection and rotational inertia neglected.

Results:

Mode number	frequency (rad./sec.)		Difference (%)
	System	Mirror Image	
1	33.44525745	33.44525745	.00000
2	108.38602303	108.38602303	.00000
3	226.13877715	226.13877715	.00000
4	386.71069206	386.71059481	.00002
5	590.10235871	590.08973387	.00211

c. System 3



system (component numbers shown)

⊗ indicates a pipe hanger

mirror images

System parameters:

Parameter	Component						
	1	2	3	4	5	6	7
diameter (in.)	2.0	2.0	2.0	H	2.0	2.0	2.0
wall thickness (in.)	.125	.125	.125	A N	.125	.125	.125
length (in.)	130.	1.40	6.0	G	60.21	1.50	6.28
radius of curv(in.)	∞	4.0	∞	E	30.	∞	4.0
incl. angle of arc	0	20°	0	R	115.°	0	90°

Intensive properties of all components except hangers:

density: 556 lbs./cu.-ft.

shear modulus: $12 \cdot 10^6$ psi

elastic modulus: $30 \cdot 10^6$ psi

Hanger spring rates:

CLX = 1000 lbs./in.

CLY = CLZ = 1000 lbs./in.

CTX = CTY = CTZ = 1000 in.-lb./rad.

End conditions:

Both ends of system fixed.

Mathematical model: Distributed mass with the effects of shear deflection and rotational inertia considered.

Results:

in-plane vibrations

Mode number	frequency (rad./sec.)		Difference (%)
	System	Mirror Image a.	
1	116.91660080	116.91660084	.00000
2	305.21721131	305.21720649	.00000
3	567.56373058	567.56397808	.00005
4	845.99432421	846.17624950	.02150
5	————	1135.09717500	———

out-of-plane vibrations

Mode number	frequency (rad./sec.) System	Mirror Image a.	Difference (%)
1	114.21818694	114.21818694	.00000
2	243.92831220	243.92831317	.00000
3	321.40891123	321.40891770	.00000
4	591.40446121	591.40449774	.00000
5	872.23102583	872.24253187	.00131
6	1138.01776773	1138.19615015	.01567
7	1571.78173050	———	———

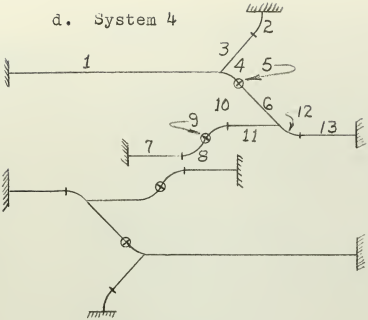
in-plane vibrations

Mode number	frequency (rad./sec.) System	Mirror Image b.	Difference (%)
1	116.91660080	116.91660080	.00000
2	305.21721131	305.21721131	.00000
3	567.56373058	567.56373058	.00000
4	845.99432421	845.9943421	.00000

out-of-plane vibrations

Mode number	frequency (rad./sec.) System	Mirror image b.	Difference (%)
1	114.21818694	114.21818694	.00000
2	243.92831220	243.92831220	.00000
3	321.40891123	321.40891123	.00000
4	591.40446121	591.40446121	.00000
5	872.23102583	872.23102583	.00000
6	1138.01776773	1138.01776773	.00000
7	1571.78173050	1571.78173050	.00000

d. System 4



system (component numbers shown)

⊗ indicates pipe hanger

mirror image

System parameters:

	Component						
Parameter	1	2	3	4	5	6	7
diameter (in.)	2.0	2.0	2.0	2.0	H	2.0	2.0
wall thickness (in.)	.125	.125	.125	.125	A	.125	.125
length (in.)	400.	39.27	100.	39.27	N	100.	100.
radius of curv.(in.)	∞	50.	∞	50.	G	∞	∞
incl. angle of arc	0°	45°	0°	45°		0°	0°

	Component					
Parameter	8	9	10	11	12	13
diameter (in.)	2.0	H	2.0	2.0	2.0	2.0
wall thickness (in.)	.125	A	.125	.125	.125	.125
length (in.)	39.27	N G	39.27	100.	39.27	100.
radius of curv.(in.)	50.	E	50.	∞	50.	∞
incl. angle of arc	45°	R	45°	0°	45°	0°

Intensive properties of all components except hangers:

density: 460 lbs./cu.-ft.

shear modulus: $12 \cdot 10^6$ psi

elastic modulus $30 \cdot 10^6$ psi

Hanger spring rates:

CLX = CLY = CLZ = 1000. lbs./in.

CTX = CTY = CTZ = 1000. in.-lb./rad.

End conditions: all main member and branch ends fixed.

Mathematical model: Distributed mass with the effects of shear deflection and rotational inertia considered.

Results:

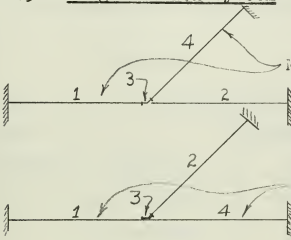
in-plane vibrations

Mode number	frequency (rad/sec.)		Difference (%)
	System	Mirror Image	
1	18.55192182	18.55189208	.00016
2	51.25211856	51.25212736	.00001
3	————	90.94757587	————
4	————	107.19459215	————

out-of-plane vibrations

Mode number	frequency (rad./sec.)		
	System	Mirror image	Difference(%)
1	14.31754957	14.31754879	.00000
2	34.45495449	34.45494654	.00002
3	41.07914530	41.07913499	.00002
4	73.06452245	73.06429798	.00030
5	115.86572509	115.86505834	.00057

E.32 Single Branch System



Case 1 (component numbers shown)
$\phi = 180^\circ$

Case 2 (component numbers shown)
$\phi = 180^\circ$

System Parameters:

Parameter	Component			
	1	2	3	4
diameter (in.)	2.0	2.0	2.0	2.0
wall thickness (in.)	.125	.125	.125	.125
length (in.)	100.	100.	.01	100.
radius of curv.(in.)	∞	∞	.01	∞
incl. angle of arc	0°	0°	45°	0°

Intensive properties of components:

density: 556 lbs./cu.-ft.

shear modulus: $12 \cdot 10^6$ psi

elastic modulus: $30 \cdot 10^6$ psi

End conditions: Both ends of main member, and end of branch, fixed.

Mathematical model: Lumped mass with the effects of shear deflection and rotational inertia neglected.

Results:

out-of-plane vibrations

Mode number	frequency (rad./sec.)		Difference (%)
	System 1	System 2	
1	49.64604938	49.64604961	.00000
2	135.52684605	135.52685066	.00000
3	269.83567590	269.83569992	.00000
4	530.72012316	530.75718901	.00698
5	675.51662199	675.57044008	.00796
6	901.52786818	901.76873827	.02671

In-plane vibrations:

Mode number	frequency (rad./sec.) System 1	System 2	Difference (%)
1	132.89829688	132.89836401	.00005
2	190.61015386	190.61009901	.00002
3	430.03806330	430.03948554	.00033
4	512.55370411	512.54463160	.00177
5	893.66594383	893.10993095	.06221
6	962.07559083	963.00709091	.09682

APPENDIX F

TRANSFER MATRIX SOURCE REFERENCE

F.1 General

Fourteen of the transfer matrices used in VIPIPE were drawn from the catalog of transfer matrices, pages 376-410, in Pestel's "Matrix Methods in Elastomechanics" [1]. Some of the transfer matrices constructed in VIPIPE are a composite of two separate transfer matrices from [1]. These composite matrices are constructed as shown in Figs. F.1-1, and F.1-2. The former shows the format of case 1 composite transfer matrices, the latter those of Case 2. In both

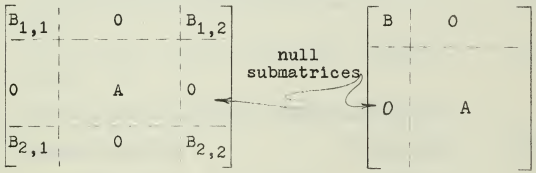


Fig. F.1-1 In-plane composite transfer matrix format

Fig. F.1-2 Out-of-plane composite transfer matrix format

cases the A matrix is of fourth order, the B matrix of second order. In Case 1 composite transfer matrices the elements of the B matrix occupy the positions shown (Fig. F.1-1).

In the following source listing, when two matrices are indicated for one subroutine, the first is the A matrix. (All component transfer matrices are constructed in subroutines.)

F.2 In-Plane Vibration Transfer Matrices

Transfer matrix	Subroutine	Source
straight section with distributed mass, shear deflection and rotational inertia	DISTM	C-3a C-1a
straight section, elastic massless field with shear deflection	SFIELD	C-3c C-1c
curved section, elastic massless field without shear deflection	CFIELD	C-7c
point mass with rotational inertia	POINT	C-3f C-1b
rigid straight section with rotational inertia, distributed mass	RIGID	C-3d C-1b
stiff corner (elbow), massless field	STIFCO	C-7f

F.3 Out-of-Plane Vibration Transfer Matrices

Transfer matrix	Subroutine	Source
straight section with distributed mass, shear deflection and rotational inertia	DISTMO	C-3a C-2a
straight section, elastic massless field with shear deflection	SFIELO	C-3c C-2c
curved section, elastic massless field with shear deflection	CFIELO	C-8c
point mass with rotational inertia	POINO	C-3f C-2b
rigid straight section with rotational inertia, distributed mass	RIGIO	C-3d C-2b
stiff corner (elbow), massless field	STIFOO	C-8f

APPENDIX G

SAMPLE PROBLEM

G.1 General Remarks

Appendix G. is intended to supplement appendix B. Included in appendix G. is an illustration of the data deck for a typical multiple branch system with hangers, and an example of the three output formats available in VIPIPE.

G.2 The Data Deck (fig. G.2-2)

The data deck as shown calls for determination of both Case 1, and Case 2 natural frequencies (IOP=0). If only Case 1 frequencies were sought, card 4 would be removed from the deck, and IOP[1] (circled field on the first data card) would be set equal to 1. Similarly, if only Case 2 frequencies were sought, cards 2 and 3 would be removed, and IOP set equal to 2.

In the sample problem configuration all boundary conditions may be specified by code (B.61), but in order to illustrate the method of programming boundary conditions for which no code is provided, the Case 1 boundary condition state vector for branch 1 is read-in directly. Note that the second field of card 2 is blank (blanks are read as zeros by the computer) and that card 3 contains the particular

[1] See appen. B., section B.71.

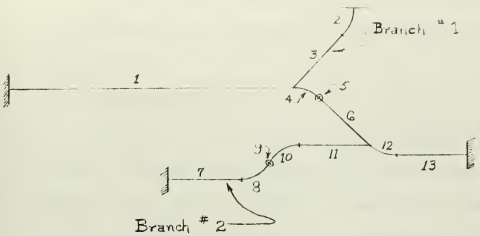


Fig. G.2-1 Multiple Branch System

```
  460.    30.00E 06 12.00E 06
    2.   .125       100.
    2.   .125 -50.        45.
    2.   .125       100.                                                  3
    2.   .125  50.        45.                                            2
                                2000.  1000.  1000.  1000.  1000.  1000.  2
    2.   .125 -50.        45.                                            2
    2.   .125       100.                                                  1
    2.   .125       100.
                                2000.  1000.  1000.  1000.  1000.  1000.
    2.   .125  50.        45.
    2.   .125       100.                                                  3
    2.   .125  50.        45.                                            1
    2.   .125       400.
  225.  45.
    1.  1.  1.  1.
        1.1.1.
    1.      1.  1.
  13 2 010.     0.  0.  5000.   .01     2.
```

④
③
②
①

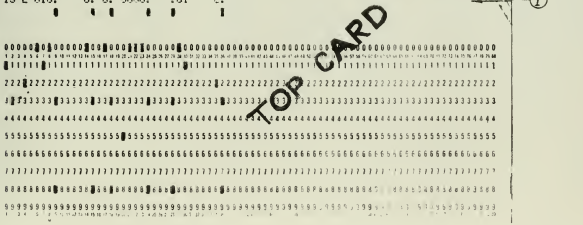


Fig. G.2-2 Sample Input Deck

boundary condition state vector.

The data deck used in Fig. G.2-2 pertains to system 4 of section E.31, appendix E. The system is illustrated in Fig. G.2-1.

G.3 Output Options

Examples of the three output formats available in VIPIPE, through specification of the value of the parameter GDABC (B.71), are included on pages G-4, and G-5. The system analyzed was system 3 of section E.31, appendix E. and is illustrated there. The problem numbers refer to the order in which the data decks are "stacked" (B.8). The heading ... "PROGRAM VIPIPE" ... etc. is printed once per run, e.g., if 4 data decks were "stacked" behind the program deck, and each asked for Case 1 and Case 2 analyses the heading would precede the print out of the results of the Case 1 analysis of system 1 only.

PROGRAM VIPIPE G. E. FINK NRA-1 FEBRUARY 1964

METHOD OF TRANSFER MATRICES. MODAL FREQUENCIES OF A PLANAR PIPING SYSTEM ARE DETERMINED BY AN ITERATIVE PROCEDURE USING THE

PROBLEM 1

IN PLANE MODE FREQUENCIES (RADIANS PER SECOND)

MODE	FREQUENCY
1	116.91660084
2	303.21726892
3	567.56429170
4	846.14045651

PROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITATION OF COMPUTER.

A DISTRIBUTED MASS APPROACH WAS EMPLOYED AND
THE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WERE CONSIDERED.

Fig. G.3-1 GDABC = 0.

PROGRAM VIPIPE G. E. FINK NRA-1 FEBRUARY 1964

METHOD OF TRANSFER MATRICES. MODAL FREQUENCIES OF A PLANAR PIPING SYSTEM ARE DETERMINED BY AN ITERATIVE PROCEDURE USING THE

PROBLEM 1

IN PLANE MODE FREQUENCIES (RADIANS PER SECOND)

MODE	FREQUENCY
1	116.91660080
2	303.21721073
3	567.56385676
4	846.05107199
5	1128.5763497?

PROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITATION OF COMPUTER.

A DISTRIBUTED MASS APPROACH WAS EMPLOYED AND
THE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WERE CONSIDERED.

INPUT DATA

COMPONENT	DIAMETER	WALL THICKNESS	LENGTH	RADIUS OF CURVATURE	INCLUDED ANGLE OF ARC	DENSITY	ELASTIC MODULUS	SHEAR MODULUS
1	2.000	.075	130.000	.000	.00	556.00	.30E+08	.12E+08
2	2.000	.075	1.396	-4.000	20.00	556.00	.30E+08	.12E+08
3	2.000	.075	4.000	.000	.00	556.00	.30E+08	.12E+08
4	2.000	.075	60.214	30.000	115.00	556.00	.30E+08	.12E+08
5	2.000	.075	1.396	.000	.00	556.00	.30E+08	.12E+08
6	2.000	.075	1.570	-4.000	90.00	556.00	.30E+08	.12E+08

HANGERS

COMPONENT	CLX	CLY	CLZ	CTX	CTY	CTZ
4	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

BOUNDARY CONDITIONS
SYS= * : : : 1: 1: 1:

Fig. G. 3-2 GDABC = 1.

PROBLEM 2

IN PLANE MODE FREQUENCIES (RADIANS PER SECOND)

MODE	FREQUENCY
1	116.9166C080
2	3C5.21721073
3	567.5639587C
4	846.05107199
5	1128.57634979

PROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITATION OF COMPUTER.

A DISTRIBUTED PASS APPROACH WAS EMPLOYED AND
THE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WERE CONSIDERED.

****

SECTION LENGTH AND SUBSECTION DATA

SECTION NUMBER	LENGTH OF SECTION (INCHES)	NUMBER OF SUBSECTIONS
1	130.00	1.
2	1.40	1.
3	6.00	1.
4	60.21	10.
5	1.50	1.
6	6.28	1.

GRAPH DATA

ITERATION NUMBER	FREQUENCY (RADIANS PER SECOND)	VALUE OF FREQUENCY DETERMINANT
1	.C10000	-.1321821E-07
2	34.275214	-.1183312E-07
3	68.540429	-.7973254E-08
4	102.805643	-.2440865E-08
5	137.070857	-.1401393E-08
6	106.522165	-.1872379E-08
7	137.346771	-.1106407E-08
8	116.941929	-.1704822E-08
9	126.679432	-.1206226E-10
10	127.679432	-.1423689E-08
11	117.368359	-.7918127E-10
12	116.940044	-.2411007E-11
13	122.399035	-.9583810E-09
14	150.669249	-.6508528E-08
15	190.929644	-.1032634E-07
16	225.194078	-.1128516E-07
17	259.458893	-.8779930E-08
18	293.723107	-.2700852E-08
19	327.987022	-.6091853E-08
20	362.252238	-.1326936E-08
21	396.516452	-.2228132E-08
22	308.884187	-.1527863E-08
23	299.251519	-.2457645E-09
24	304.004677	-.1338709E-08
25	308.287823	-.7679794E-09
26	304.417986	-.1194933E-08
27	306.574562	-.1352509E-09
28	311.242830	-.1524377E-08
29	461.883318	-.5435838E-07
30	612.523800	-.2770824E-07
31	476.947367	-.3347918E-07
32	524.220701	-.8601268E-08
33	627.986852	-.4175854E-07
34	659.799592	-.1483547E-08
35	523.455988	-.2421699E-08
36	523.926968	-.2421699E-08
37	582.198758	-.5524957E-08
38	595.448654	-.1290914E-08
39	596.886684	-.4548103E-08
40	590.390157	-.7194103IE-09
41	591.097672	-.2191313E-08
42	606.860908	-.4326921E-09
43	569.211666	-.1013330E-08
44	575.958952	-.5291523E-08
45	769.819277	-.7159952E-07
46	905.113588	-.2523804E-06
47	806.824002	-.5273567E-07
48	911.762661	-.1237376E-06
49	817.117068	-.1405917E-07
50	859.384881	-.9402923E-07
51	855.126801	-.3345158E-08
52	868.798467	-.1305590E-07
53	825.188267	-.3055902E-07
54	838.305600	-.1210746E-07
55	851.422933	-.8569522E-08
56	859.417333	-.9935510E-08
57	846.171999	-.1746229E-09
58	840.213199	-.9205951E-08
59	843.552532	-.3725290E-08
60	846.651866	-.1106153E-08
61	846.982663	-.1469370E-07
62	1077.752933	-.1497409E-06
63	1300.543204	-.1430515E-05
64	1100.051470	-.1490116E-06
65	1211.476896	-.6853938E-07
66	1119.524512	-.2235387E-07
67	1116.886843	-.2384185E-06
68	1116.746843	-.8195638E-07
70	1144.528965	-.8940696E-07
71	1119.524512	-.5215604E-07
72	1133.449872	-.7450232E-07
73	1120.917951	-.5960464E-07
74	1127.080132	-.2980232E-07
75	1134.842312	-.4470348E-07
76	1120.574550	-.0000000E+00
77	1124.162554	-.2980232E-07
78	1133.073175	-.2980232E-07
79	1135.611759	-.1490116E-07
80	1264.027381	-.1490116E-06
81	1264.027381	-.4470348E-07
82	1300.253603	-.4656792E-06
83	1496.249114	-.5960464E-07
84	1494.129115	-.0000000E+00
85	1503.769819	-.3814697E-05

INPUT DATA

COMPONENT	DIAMETER	WALL THICKNESS	LENGTH	RADIUS OF CURVATURE	INCLUDED ANGLE OF ARC	DENSITY	ELASTIC MODULUS	SHEAR MODULUS
1	2.000	.075	130.000	.000	.00	556.00	.30E+08	.12E+08
2	2.000	.075	1.396	4.000	20.00	556.00	.30E+08	.12E+08
3	2.000	.075	6.000	4.000	90.00	556.00	.30E+08	.12E+08
4	2.000	.075	60.210	-30.000	115.00	556.00	.30E+08	.12E+08
5	2.000	.075	1.500	.000	.00	556.00	.30E+08	.12E+08
6	2.000	.075	6.283	4.000	90.00	556.00	.30E+08	.12E+08

HANGERS

COMPONENT	CLX	CLY	CLZ	CTX	CTY	CTZ
4	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

BOUNDARY CONDITIONS

SYS : : : : : : :

Fig. G.3-3 GDABC = 3.

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